ENDOGENOUS GROWTH IN A MODEL OF DYNAMIC SCORING WITH PUBLIC CAPITAL

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Abstract

This paper examines the long-term effects of tax cuts on tax revenue in a model that allows for transitional dynamics. Whereas previous dynamic scoring models have assumed an exogenous, or given, growth rate, this paper changes the production function by altering the basic neoclassical Ramsey model in order to obtain endogenous growth. The transitional dynamics come from the inclusion of private and public capital as stock variables, rather than flow. The model in this paper is designed to find the revenue feedback effects from reductions in the capital tax rate. The study uses this model to explore the possibility of "feasible tax cuts": those which would generate sufficient revenue feedbacks from increased economic growth to compensate for the initial revenue loss. These are measured as having feedback effects equal to the present value of future exogenous government expenditure. This study finds that a range of feasible capital tax cuts is able to exist in an economy with an optimal fiscal environment.

KEYWORDS: (Endogenous Growth, Dynamic Scoring, Tax Revenue)

ON MY HONOR, I HAVE NEITHER GIVEN NOR RECEIVED UNAUTHORIZED AID ON THIS THESIS

Signature

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CHAPTER 1

INRODUCTION

The economic impact of tax cuts has been the center of many heated political debates recently, as disputes have arisen around the validity of various methods of measuring the impact of the 2001 and 2003 Bush tax cuts. Ever since their proposal, the potential effects of the tax cuts on the economy have been a point of disagreement. The ability of tax cuts to stimulate a stale economy is a staple of supply-side economic theory. Supply-side economists argue that tax cuts spur consumption and private investment, bringing about significant economic growth[1]. Opponents of tax cuts take the stance that lower taxes mean lower government revenue[2]. This in turn leads to a decline in government investment and services, which lowers the social and economic welfare of the American public.

The tax cuts put into place by the Bush administration in 2001 and 2003 are again under debate as they draw near their expiration dates. The Economic Growth and Tax Relief Reconciliation Act (EGTRRA), enacted in 2001, lowered marginal tax rates on individual income and is set to expire in 2008. The Jobs Growth and Tax Relief Reconciliation Act (JGTRRA), enacted in 2003, lowered rates on capital gains and

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[1] Tracy L. Foertsch and Ralph A. Rector, *The 2001 and 2003 Bush Tax Cuts: Economic Effects of Permanent Extension* (Washington, DC: The Heritage Foundation, 2007), 1.

[2] Deborah Solomon, "For Bush, Saving Tax Cuts is a Tough Balancing Act," *The Wall Street Journal,* February 3 2007, sec. A, p. A2.

dividends taxes and accelerated the provisions set forth in the 2001 tax cuts; it is set to expire in 2010[3]. In the past few months, movements have been made to make the tax cuts permanent. The most asked question is what permanent tax reductions would do to the growing national deficit. In the current state of the budget, many public services are operating on limited funds, and as such are hesitant to back a policy that might only aggravate those budget constraints. The reasoning behind these tax cuts is that reductions in tax rates increase the welfare of the American citizens by increasing their personal wealth and spurring the growth of the economy[4]. However, the validity of the ability of the provisions in the tax cuts to increase the wealth of the individual citizen has been under fire from many Democratic groups. The Joint Economic Committee Democrats (JECD) published a study in 2004 that showed the tax cuts as having greatly benefited higher-income households but not low- or middle-income households. Using data from the Congressional Budget Office, the committee found a huge disparity between the benefits of the tax cuts to high-income families and middle- and lower-income families. The study calculated that while the percent change in after-tax income for the top 20% of households is 5.2%, it is 2.3% for the middle 20%, and for the bottom 20%, only 1.6%. This, it argues, has lead to a disproportionate growth in pre-tax income for high-income households, aggravating the nation's income gap. The JECD study also addressed the cost of the tax cuts, faulting the current administration of neglecting to take into consideration the long-term effect on the private citizens who will carry the cost burden, saying, "The Administration has pushed those costs into the future by increasing

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[3] Foertsch and Rector, "The 2001 and 2003 Bush Tax Cuts: Economic Effects of Permanent Extension," 1.

[4] "Fact Sheet: President Bush Taking Action to Strengthen America's Economy," *The White House* (2003), [journal on-line]; available from http://www.whitehouse.gov/news/releases/2003/01/20030107.html; Internet; accessed March 28, 2007.

government borrowing to finance the tax cuts, but eventually that debt must be paid either through spending cuts or increased revenues". The data used suggested that when the cost of repaying the tax cuts is included, the bottom four-fifths of households will lose more than they gain[5].

In a recent budget meeting, Democratic Senator Kent Conrad of North Dakota, chairman of the Senate Budget Committee, vocalized a fear that government services, such as retirement programs, will effectively cease to exist due to lack of funding if taxes are not raised[6]. In response, US Treasury Secretary Henry Paulson defended the tax cuts on the basis that extended cuts will continue to preserve a strong economy, and crediting the current state of the economy to the 2001 and 2003 tax cut acts. He argued that by increasing private spending, they created more jobs, raised wages, and through the ensuing increase in personal wealth, increased the tax base, helping to reduce the deficit rather than increase it[7].

A 2007 paper published by The Heritage Foundation studied the effects of permanently extending the 2001 EGTRRA and 2003 JGTRRA acts using data from the Center for Data Analysis. The study found in a dynamic macroeconomic analysis that a permanent extension of the tax cuts would create an average of over 700,000 jobs per year from 2011 through 2016. In this same time period, real GDP would increase an average of $76.6 billion annually, and by 2016 real GDP would be over $60.2 billion higher than the real GDP projections for 2016 that assume the tax cuts expire as planned

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[5] Pete Stark, *New CBO Analysis Confirms that the Bush Tax Cuts are Skewed Toward the Rich* (Washington, DC: Joint Economic Committee Democrats, 2004), 1,

[6] Solomon, "For Bush, Saving Tax Cuts is a Tough Balancing Act," A2.

[7] Elizabeth Price, "Paulson Backs Bush Tax Cuts in Budget Defense to Congress," *The Wall Street Journal,* February 6 2007,

in 2008 and 2010. With permanent tax cuts, real disposable income would rise by $197 billon on average per year from 2011 through 2016 and the savings rate would increase 0.8% per year. The study found that the greatest positive effects of a permanent extension come from the lower taxes on capital gains, dividend income, and ordinary income. In a dynamic projection, the Heritage Foundation finds the budget extension to generate roughly $295.5 billion in revenue feedbacks to the government. They argue that "cuts in marginal tax rates both increase the after-tax wage rate and lower the cost of capital. They therefore tend to encourage individuals to work more and businesses to invest. Increases in labor supply, saving, and the domestic capital stock follow"[8].

The Wall Street Journal quotes budget expert Stan Collender in a recent article as saying, "Depending on how you do the math, he says, there's 'either a big surplus coming or a big deficit coming'"[9]. This sentiment is echoed by many as a new process for measuring these effects is beginning to draw more and more attention as it develops. Dynamic scoring measures the effects of tax cuts on tax revenue by examining long-run dynamics in a shocked model of the economy. Many economists recognize the ability of a reduction in tax rates to cause growth, though the extent to which a tax cut is able to finance itself is under debate.

This paper will build off of current dynamic scoring models by endogenizing economic growth in a model with public capital.

The remainder of this paper is organized as follows. Chapter 2 provides a framework for the analysis by reviewing the literature on endogenous growth, public capital and dynamic scoring. Chapter 3 develops the endogenous growth model with

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[8] Foertsch and Rector, "The 2001 and 2003 Bush Tax Cuts: Economic Effects of Permanent Extension," 1.

[9] Solomon, "For Bush, Saving Tax Cuts is a Tough Balancing Act," A2.

public capital, finding the transition paths for tax cuts and measuring the feedback effects. Chapter 4 analyzes the model using dynamic Laffer curves and comparative statics. Chapter 5 concludes.

CHAPTER 2

LITERATURE REVIEW

The purpose of this chapter is to introduce the concept of endogenous growth and examine the existing literature on the topic.

2.1 Endogenous Versus Exogenous Growth

Exogenous growth in a model of long-term growth can yield very misleading results. An exogenous variable is one given as a value deemed appropriate by the author. While this is fitting for some variables, such as tax rates, it is not for others; exogenous growth rates can be very inaccurate. Realistically, growth is not a constant variable. There are many factors that affect growth rates and can change them intertemporaneously in a dynamic setting, but exogeneity prohibits that. On an intuitive level, growth as a given does not make sense. Endogeneity makes a variable in a model to rely on the other variables present. An endogenous variable moves in relation to changes in its surrounding variables, making it much more appropriate than exogeneity in a growth model. Futagami et al (1993)[1] argue that endogenous models are much better suited to observe the effects of taxation on the rate of growth since in the long-run, "fiscal policy can affect only the

[1] Koichi Futagami, Yuichi Morita, and Akihisa Shibata, "Dynamic Analysis of an Endogenous Growth Model with Public Capital," *Scandinavian Journal of Economics* 95 (1993): 607.

per-capita *levels* of capital and income"[2]. Following this reasoning, the model in this paper employs endogenous growth.

The creation of endogenous growth models often begins with a Cobb-Douglas production function, $Y = AK^{\alpha}L^{\beta}$. If α and β sum to one, the model is endogenous, meaning capital, K, and labor, L, have constant marginal returns when taken together, even though they demonstrate diminishing returns separately. This ability of capital and labor to have constant returns over the long-run is its distinguishing factor, since while exogenous growth models can show transition paths in dynamic settings, endogenous growth models, due to their ability to eliminate diminishing returns, are more stable and do not deteriorate over time, making them superior to exogenous growth when studying long-term dynamic growth (Barro and Sala-i-Martin 1995)[3].

The majority of the works consulted in this paper begin endogenous models with a basic Ramsey AK production function: $y = f(k) = Ak$, where y is output per capita, A is a constant measure for technology, and k is capital per capita, defined loosely to include both physical and human capital. Many papers combine physical and human capital into this one stock variable k, since whether they are separate or combined, they must be perfect substitutes for each other (their exponents must sum to 1) in order to ensure that they move in concert to avoid the diminishing returns that set in if they do not (Ireland, 1993)[4]. Because their model has only one stock variable, k, it does not have transitional dynamics; the model is always on a balanced growth path where the growth rates of

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[2] Barro, "Government Spending in a Simple Model of Endogenous Growth," S103-S125.

[3] Robert J. Barro and Xavier Sala-I-Martin, *Economic Growth,* 2d ed. (Cambridge, MA: MIT Press, 2004), 654.

[4] Peter N. Ireland, "Supply-Side Economics and Endogenous Growth," *Journal of Monetary Economics* 33 (06 1994): 559-571.

output and capital are constant. The model is then relaxed and adjusted as needed, while still upholding the necessary components for endogenous growth. Since growth in the basic Ramsey model is determined solely by demographic and technological changes (changes in A), Turnovsky (1996) poses that it is not appropriate in studies such as those analyzed in this paper, since in the long run dynamic growth models created here, it is the effect of fiscal policy on growth that is being observed (Turnovsky (1996)[5]; Barro (1990)[6]).

2.2 Endogenous Growth and Public Capital

The seminal work on endogenous growth and public capital is Barro (1990). In his paper, Barro modifies the Ramsey Ak model to include tax-financed government services. This is incorporated as a capital-like input that affects production and utility, and is used as a flow rather than a stock; he retains the k variable as an all-encompassing measure of capital[7]. Subsequent economists have modified this to separate physical and human capital and create a two stock model (Barro (1990)[8]; Lucas (1968)[9]; Rebelo (1991)[10]). The benefit of this separation is that it allows for transitional dynamics, though one must

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[5] Stephen J. Turnovsky, "Fiscal Policy, Adjustment Costs, and Endogenous Growth," *Oxford Economic Papers* 48 (07 1996): 361.

[6] Robert J. Barro, "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy* 98 (10 1990): S103-S125.

[7] This approach makes more sense when assuming constant returns to capital since human and physical capital are not perfect substitutes, meaning they would have to move together in a model to avoid diminishing returns to either separately.

[8] Ibid.

[9] Robert E. Lucas Jr., "On the Mechanics of Economic Development," *Journal of Monetary Economics* 22 (07 1988): 3-42.

[10] Sergio Rebelo, "Long-Run Policy Analysis and Long-Run Growth," *Journal of Political Economy* 99 (06 1991): 500-521.

keep in mind the Ramsey model's stipulation that all variables used must have constant returns to scale. In a two-variable model, this means that the exponents on all variables must sum to one to ensure that every variable is growing at the same rate; in a one-variable model, one need not worry since a single variable will grow at the same rate as itself. It is important to note that a two stock model is unnecessary in Barro's paper since it studies capital accumulation with constant returns to scale in an economy assumed to be in a constant state of steady growth, rather than the transition paths from an arbitrary starting point to a new steady growth rate of the ratio of physical capital to human capital obtained by using more than one stock variable[11].

Barro includes a public sector by assuming that all government expenditure is in the form of productive investment that increases the productivity of private capital, k, and thereby increases the growth rate. He then replaces the traditional augmenter of growth, technology, A, with government expenditure on public services, g. This idea of a complementary relationship between public and private capital was popularized by Aschauer (1988).[12] Note that g is modeled as a flow, not a stock. This can be explained by viewing government services as government purchases of final goods which are used to provide private capital-enhancing services to all and are assumed not to be subject to congestion; nothing is produced or accumulated by the government. Because of externalities that arise from public expenditures and the taxes by which they are financed, privately determined values of savings and economic growth may be suboptimal

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[11] Ibid.

[12] David A. Aschauer, *Is Public Expenditure Productive? Manuscript,* (Chicago: Federal Reserve Bank Chicago, 1988)

(Turnovsky 1998)[13]. Barro's production function is now changed to make output a function of the ratio of public services to private capital: $y = \Phi(k,g) = k * \Phi (g/k)$. As stated above, $\Phi (g/k)$ has taken the place of A as the augmenter of growth, essentially making A a function of g/k: $A = \Phi (g/k)$. The original consumption growth model changes as well with this substitution so that the model now includes a flat tax rate, τ, which yields tax revenue (by which all government expenditures are funded) when paired with y, as well as serving as the elasticity of output with respect to g. This substitution for A is now the post-tax marginal product of capital, which is assumed to be the factor of the model that produces growth instead of technological advancement. The economy is on its steady state growth path when the government has set τ constant, so that all variables, y, k, g, and c, grow at the same equilibrium rate[14][15].

Futagami et al (1993)[16][17] builds upon Barro's model by integrating public capital stock rather than public investment flow into the model. This addition increases the complexity of the model and gives it transitional dynamics. Futagami rationalizes this by observing that in nature, public investment often goes towards stock variables such as highways and sewer systems that are used by private agents. It is also public capital, rather than investment, that most enhances the productivity of private capital[18]. The

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[13] Stephen J. Turnovsky, "Fiscal Policy, Elastic Labor Supply, and Endogenous Growth," *Journal of Monetary Economics* 45 (02 2000): 185-210.

[14] The dynamics are the same as the Ak model since the fundamental model is not changed, one variable is simply substituted out for an equal variable.

[15] Barro, "Government Spending in a Simple Model of Endogenous Growth," S103-S125.

[16] Futagami et al (1993) will hereon be referred to as Futagami

[17] Futagami, Morita, and Shibata, "Dynamic Analysis of an Endogenous Growth Model with Public Capital," 607.

[18] See also Aschauer (1988)

model is constructed by inserting a new production function into Barro's standard budget constraint and consumption functions: government expenditures are equal to tax revenue, R, so can be re-written as $g = R = \tau y = \tau \Phi (g/k)$, where τ is a flat income tax rate, and $\Phi (g/k) = y$. The inclusion of public capital as a stock yields results slightly different from those of Barro (1990). Futagami finds the tax rate that maximizes economic growth is obtained when the tax rate equals the output elasticity of public capital. However, the growth maximizing tax rate is found not to equal the welfare maximizing rate, rather the optimal tax rate is lower than the growth maximizing rate.

2.3 Endogenous Growth and Dynamic Laffer Analysis

Laffer theory states that in certain economic situations, tax cuts can, through growth, pay for themselves in entirety. This is illustrated as a graph of tax revenue versus tax rates and is shaped as a skewed inverted hyperbola. The x intercepts mark the points where tax rates equal 0 and 100, both of which yield no tax revenue. The highest point of the graph is the tax rate that yields the maximum tax revenue. To the left of this point, an increase in taxes will increase tax revenue, but to the right side of this point, a *decrease* in tax rates will increase tax revenue. The importance of this to dynamic scoring is that when on the right side of the graph, a decrease in tax rates will spur enough economic growth to pay for itself in entirety. This is explained by the increase in personal wealth that results from lower taxes which increases the tax base, so while the tax rate is a smaller percentage of the tax base, the larger size of the tax base will compensate for the rate reduction and yield the same dollar amount of tax revenue.

The Laffer Curve has been conditionally accepted by economists, though the extent to which a tax cut is self-financing is widely disputed. Novales and Ruiz (2002) study the possibility of "feasible" tax cuts based on this theory, stating that there exists a design for a deficit-financed tax cut such that, all other taxes constant, "the present value of future revenues [may be] higher than that of future expenditures"[19]. The peak of the Laffer Curve is the point at which the tax rate on human capital, physical capital, and consumption produce the maximum level of tax revenue. Novales and Ruiz (2002)[20] and Ireland (1994)[21], however, seek the point on the Laffer curve that yields the lowest tax rate able to support the exogenous flow of government expenditures[22]. By including two stock variables, human and physical capital accumulation[23], Novales and Ruiz are able to study the transition path between the two balanced growth paths and compare the short-run and long-run effects of the tax cuts. They do this by implementing a deficit-financed cut in income taxes. Tax revenue, physical and human capital, and consumption originally grow at the same rate as government expenditures, g. However, the tax cut changes all growth rates save government expenditures. Government expenditures are now funded by government bond income, and thus are able to grow at the constant pre-tax rate. This difference in growth rates between variables is what yields transitional

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[19] Alfonso Novales and Jesús Ruiz, "Dynamic Laffer Curves," *Journal of Economic Dynamics & Control* 27 (12 2002): 181.

[20] Ibid.

[21] Ireland, "Supply-Side Economics and Endogenous Growth," 559-571.

[22] Government expenditures in the Novales and Ruiz study are assumed to be unproductive. That is, public expenditures do not affect the productivity of private capital or the marginal utility of consumers.

[23] Ireland (1994) performs a similar study, but with only one stock variable, capital, defined broadly to encompass physical and human capital.

dynamics. Eventually the economy reaches a new steady state at which the growth rates are again all equal.

The conclusions from this study differ substantially from those found in non-dynamic studies. As to be expected, tax cuts are found to have a positive effect on the rate of economic growth, though a cut in one tax while keeping all others constant is found to be possible only if debt is issued to help finance the areas where tax rates remain constant since short-term tax revenue declines and is not able to support government expenditures without another revenue source. The maximum feasible tax cut is also found to yield the largest possible increase in welfare.[24] The dynamic element of this study appears in the short-term effects of a reduction in tax rates. Novales and Ruiz conclude that, "impulse response functions following a policy intervention often change their sign over time; aggregate budget and welfare effects can be substantially different from the long-run effects"[25]. This means that while immediately after a tax cut government revenue will drop, it then begins to increase again slowly as the economy grows until it rests at its new steady state growth rate: the short term and long term effects have opposite signs. Reductions in income and capital taxes were found to have roughly equal effects[26], whereas consumption tax cuts were found to be infeasible in all cases.

2.4 Endogenous Growth and Dynamic Scoring

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[24] However, numerous other studies have found this to be false. See Barro (1990); Jones, Manuelli and Rossi (1993); Bruce and Turnovsky (1999).

[25] Novales and Ruiz, "Dynamic Laffer Curves," 181.

[26] In most cases, the largest feasible cut in income taxes raises welfare more than the largest feasible cut in capital taxes. However, in economies where consumption is highly preferred over leisure, capital tax cuts are found to have larger effects that income tax cuts. The same is also found when income taxes are substantially lower than capital taxes.

When making fiscal policy decisions, tax cuts are generally viewed as proportional cuts in tax revenue since the method used when analyzing the effects of tax cuts does not take macroeconomic dynamics into account. Tax cuts in this model of "static scoring" are assumed to have no feedback effect into government revenue.

Recently, many economists have become increasingly stronger proponents of "dynamic scoring", a tax analysis method acknowledges the feedback effects generated by the increased tax base from tax cuts. Dynamic scoring theory is based on the ability of tax cuts to stimulate a stagnant economy. The Congressional Budget Office (CBO) currently uses static scoring methods to analyze possible tax cuts[27]. The results from dynamic scoring models differ greatly from those generated by static scoring and as such, any discussion concerning the possibility of changing the CBO's methodology will have political implications as well as economic. Dynamic scoring models are based off of the basic neoclassical Ramsey growth model (1928)[28] and include any combination of variables such as private capital, both human and physical, government consumption and investment expenditures, government issued bonds to finance deficit spending, labor-leisure choices, or anything deemed appropriate by the author. The question most debated among economists is to what extent a tax cut can pay for itself through growth. While the Laffer curve would argue 100%, some economists believe this to be unrealistic and have developed dynamic scoring models to generate estimates that are below 100% yet still highly significant.

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[27] Pete Stark, *New CBO Analysis Confirms that the Bush Tax Cuts are Skewed Toward the Rich* (Washington, DC: Joint Economic Committe Democrats, 2004), 1.

[28] Frank P. Ramsey, "A Mathematical Theory of Saving," *Economic Journal* , no. 38 (December 1928): 543-559.

Bruce and Turnovsky (1999)[29] created a dynamic scoring model, devising a two-sector, inelastic labor supply model to study the effect of government policy on long-term budget. In the study, public expenditures are broken down into government consumption, defined as a drain on the economy, and government investment that, similar to Aschauer (1988)[30], increases the productivity of private capital. The study concludes that simultaneous reductions in income tax rates and government consumption have significant implications for the long-term fiscal balance. While the feedback is not 100%, it is found to be magnified if the cuts are more heavily weighted towards government consumption expenditures. The size of the behavioral intertemporal elasticity of substitution in this model is crucial in deriving the amount of feedback generated.

Mankiw (2004)[31] studies the long-term effects of cuts in income, capital, and lump sum taxes. He adjusts the Ramsey model to observe the behavioral intertemporal elasticity of substitution of reductions in each cut. He concludes that in a model with an elastic labor supply, a cut in income tax rates is significantly more effective than a cut in capital tax rates, and will immediately pay for 10% of itself through higher growth, eventually financing roughly 50% of itself. However, in a model with an inelastic labor supply, he found exactly the opposite, since a reduction in income tax rates cannot affect an inelastic labor supply.

Chapters three and four will develop an endogenous growth dynamic scoring model with public capital and an inelastic labor supply. Chapter five will conclude.

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[29] Neil Bruce and Steven J. Turnovsky, "Budget Balance, Welfare, and the Growth Rate: "Dynamic Scoring" of the Long-Run Government Budget," *Journal of Money, Credit & Banking* 31 (05 1999): 162-186.

[30] Aschauer, "Is Public Expenditure Productive? Manuscript,"

[31] N. Gregory Mankiw and Matthew Weinzierl, "Dynamic Scoring: A Back-of-the-Envelope Guide," *Journal of Public Economics* 90, no. 8-9 (09// 2006): 1415-1433.

CHAPTER 3

THE MODEL

The purpose of this chapter is to develop an endogenous growth model for dynamic scoring that includes public capital. The feedback effects and Laffer curves will be analyzed and discussed.

3.1 The Inelastic Labor Supply Model

The Ramsey (1928)[1], neoclassical growth model, a staple of macroeconomic growth dynamics, is used by Mankiw and Weizierl (2006)[2] in their work on dynamic scoring, as well as by Barro (1990)[3] in his paper on endogenous growth. The Ramsey model is desirable because of its elegant simplicity and the ease with which it is modified and relaxed. This paper modifies the Ramsey model to include public capital. As per Aschauer (1988)[4], government expenditure on infrastructure, or public capital, is assumed to be a positive input into private capital, increasing its productivity. In this

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[1] Frank P. Ramsey, "A Mathematical Theory of Saving," *Economic Journal* , no. 38 (December 1928): 543-559.

[2] N. Gregory Mankiw and Matthew Weinzierl, "Dynamic Scoring: A Back-of-the-Envelope Guide," *Journal of Public Economics* 90, no. 8-9 (09// 2006): 1415-1433.

[3] Robert J. Barro, "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy* 98 (10 1990): S103-S125.

[4] David A. Aschauer, *Is Public Expenditure Productive? Manuscript,* (Chicago: Federal Reserve Bank Chicago, 1988)

model the government is also assumed to run a balanced budget. All government action is taken by the representative household to be exogenous, and in a decentralized closed economy aggregate output, Y, is produced such that

$$Y = F(K,g_I) = K^{\beta}N^{1-\beta}g_I^{1-\beta}, \tag{1}$$

where K is aggregate private capital, g_I is per capita public capital, and N is population. β measures the output elasticity of K, N and g_I. The model is simplified by rewriting the variables in per capita terms: y = Y/N, c = C/N, *i* = I/N, $g_I = G_I/N$, and k = K/(AN)[5], where C is consumption. The production function in per capita terms is now:

$$y = F(k,g_I) = k^{\beta}g_I^{1-\beta}. \tag{2}$$

Given constant returns to scale, profits are exhausted by capital and productive government investment. The production function satisfies the Inada conditions: $f_i \rightarrow \infty$ as *i* approaches zero, and $f_i \rightarrow 0$ as *i* approaches infinity for *i* = k, g_I. Because g_I is taken as a stock, including it in the production function generates the dynamic growth conditions $\dot{k}/k$, $\dot{g}_I/g_I$ and $\dot{c}/c$; each of these shows how a variable moves over time as it travels from one steady state to the next. We will see how they are found in section 3.3. k, g_I, and c are later converted into the ratios: $x = g_I/k$, $z = c/k$, so that $\dot{x}/x = \dot{g}_I/g_I - \dot{k}/k$, $\dot{z}/z = \dot{c}/c - \dot{k}/k$ characterize the steady-growth equilibrium. Ratios must be used in order to incorporate all three variables into a two equation system. When public and private

[5] As mentioned earlier in this paper, A is a measure for technology that in the basic Ramsey model is assumed to be the only means for inducing economic growth. Since N is population, and the labor force is constant, AN measures effective labor.

capital levels and consumption are shocked from a change in tax rates, the graphs of x and y will show the economy move from one steady growth path to another. The growth rates $\dot{k}/k$, $\dot{g}_I/g_I$, and $\dot{c}/c$ will change at different rates when initially shocked, then converge to the new steady growth rate found previously when $\dot{x}/x = \dot{z}/z$. This will be discussed in more detail later in Sections 3.3 and 3.4. Competitive markets now have the input demands

$$r = \beta k^{\beta-1} g_I^{1-\beta} \tag{3}$$

$$w = (1-\beta) k^{\beta} g_I^{1-\beta} \tag{4}$$

where r is the rental rate of capital, and w is the wage rate.

3.2 Government Budget Constraint

Government expenditure is comprised of government investment in public infrastructure, g_I, and residually determined government transfers to the household, g_T. Expenditures are financed in whole by tax revenue, R. Tax revenue is amassed through labor taxes, $t_w whN$, and capital taxes, $t_k rkN$. In per capita terms, the government budget constraint is

$$t_w wh + t_k rk = i_g + g_T,$$

where h represents hours worked. H will be normalized to one in this model, creating an inelastic labor supply. Since h cannot change the labor force cannot change, meaning that changes in tax rates and wage rates will not affect the size of the labor force. In a model with an elastic labor supply, h would represent a labor-leisure choice. Its inclusion in the budget constraint would create a 2-sector model. H would then be carried through the rest of the model like k, c, and g_I.

Similar to private capital, per capita public capital is assumed to depreciate at the rate δ, such that $I_g = \dot{g}_I + \delta g_I$. In this paper, δ is normalized to one, eliminating the effects of deprecation. The government allocates funding to expenditures in the manner

$$i_g = s(t_w wh + t_k rk)$$

$$g_T = (1-s)(t_w wh + t_k rk).$$

Fiscal policy is restricted to the adjustments of s, t_k, and t_w. As such, the government cannot directly control the levels of expenditure, only the percentage of revenue allocated to various expenditures. Budget balance is maintained through the management of these fiscal parameters.

3.3 Household Utility and Budget Constraint

The economy is characterized by an infinitely-lived representative household maximizing utility in terms of consumption, C, in the isoelastic form,

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma},$$

where the parameter γ is the inverse of the intertemporal elasticity of substitution. The household aims to maximize the summation of discounted utility over an infinite time period given by,

$$\int_0^{\infty} e^{-\rho t} \frac{c^{1-\gamma}}{1-\gamma}\, dt. \tag{5}$$

The discount factor, $e^{-\rho t}$, denotes the rate of time preference, ρ. The representative household must maximize its utility subject to its budget constraint and the fiscal policies currently in place. Per capita disposable income is divided between consumption and investment in private capital, I_k.

$$t_w wh + t_k rk = i_g + g_T$$

$$wh + rk = c + I_k + i_g$$

$$I_K = \dot{k} + \delta k$$

$$wh + rk = c + \dot{k} + \delta k + t_k rk + t_w wh - g_T$$

$$(1 - t_w)w + (1 - t_k)rk + g_T = c + \dot{k} + \delta k$$

$$r = \beta k^{\beta-1} g_I^{\,1-\beta}$$

$w = (1-\beta)k^{\beta-1}g_I^{1-\beta}$.

From here on, hours worked, h, will be discluded from the model in order to retain an inelastic labor supply.

With these constraints, a Hamiltonian equation may be formed:

$$H = e^{-\rho t}\frac{c^{1-\gamma}}{1-\gamma} + \lambda_1[(1-t_w)w + (1-t_k)rk + g_T - c - \delta k] \quad (6)$$

Partial derivatives of each variable are taken

$$\frac{\partial H}{\partial c} = 0 \;\Rightarrow\; e^{-\rho t}c^{-\gamma} = \lambda \;\Rightarrow\; \dot{\lambda} = -\rho e^{-\rho t}c^{-\gamma} - e^{-\rho t}\gamma c^{-\gamma-1}\dot{c} \quad (7)$$

The time derivative of (7) gives

$$\frac{\dot{\lambda}_1}{\lambda_1} = -\gamma\frac{\dot{c}}{c} - \rho \quad (8)$$

$$\frac{\partial H}{\partial k} = \dot{\lambda} \;\Rightarrow\; -\lambda[(1-t_k)r - \delta] = \dot{\lambda} \quad (9)$$

The time derivative of (9) gives

$$\frac{\dot{\lambda}_1}{\lambda_1} = -[(1-t_k)r - \delta] \quad (10)$$

Combining (8) and (10) gives

$$\frac{\dot{c}}{c} = \frac{1}{\gamma}[(1 - t_k)r - \delta - \rho] \quad (11)$$

$$\frac{\dot{g}_I}{g_I} = sR/g_I - \delta g_I/g_I \quad (12)$$

The nature of this model precludes it from having a traditional steady state. This model is created to measure growth, since the objective of this paper is to study the addition of endogenous growth to the dynamic scoring model. Because of this, steady state levels of private and public capital are superfluous; the new steady state growth rate, or balanced growth equilibrium is the object of study. When the growth rates of consumption, private capital, and public capital are equal, the balanced growth equilibrium has been reached. The growth rates of consumption, private capital, and public capital are derived from the accumulation rates of c, k, and g, found from the first order conditions of the maximization problem. At the balanced growth equilibrium, these growth rates are all equal,

$$\Phi = \dot{c}/c = (1/\gamma)\ [(1\text{-}t_k)r - \delta - \rho]$$

$$\Phi = \dot{k}/k = (1\text{-}t_w)w/k + (1\text{-}t_k)r + [(1\text{-}s)(t_w w + t_k rk)]\ /k - c/k - \delta$$

$$\Phi = \dot{g}_I/g_I = s(t_w w + t_k rk)/g_I - \delta g_I/g_I.$$

Rules are now made: $g_P = g_I/k$ and $r = \beta k^{\beta-1}(g_P k)^{1-\beta}$ => $r = \beta A x^{1-\beta}$ in order to obtain one consolidated equation for growth:

$$\Phi = (1/\gamma)\,[(1\text{-}t_k)\beta A x^{1-\beta} - \delta - \rho],\;\; x^{1-\beta} = k^{\beta-1} g_I^{\,1-\beta}$$

Notice that t_w does not affect the growth rate. This is because the model constructed here has an inelastic labor supply. Since the labor market cannot change, the theory that an increase or decrease in labor taxes factors into an agent's labor-leisure choice and changes its hours worked to maximize utility is extraneous.

3.4 Finding Balanced Growth Paths

Make new variables $x = g_I/k$, (note that $x = g_P$), $z = c/k$, such that $\dot{x}/x = \dot{g}_I/g_I - \dot{k}/k$, $\dot{z}/z = \dot{c}/c - \dot{k}/k$[6]. Ratios must be used here to consolidate all three variables into a two equation system. The intersection of $\dot{x}/x$ and $\dot{z}/z$ before the shock is the original steady growth rate. The shock from a tax rate change shifts $\dot{x}/x$ and $\dot{z}/z$ so that the new steady growth equilibrium is shown as the point where the new growth rate $\dot{x}/x$ equals the new growth rate $\dot{z}/z$, at the new intersection. To solve for the steady state ratios x and y, the growth rates are set equal: $\dot{x}/x = \dot{z}/z$. Assume now that $\delta = 0$ and $\gamma = 1$.

$$\dot{x}/x = \dot{g}_I/g_I - \dot{k}/k = s(t_w w + t_k r k)/g_I - [(1\text{-}t_w)w/k + (1\text{-}t_k)\beta\, g_P^{\,1-\beta} + (1\text{-}s)(t_w w + t_k r k)/k - z] \quad (13)$$

[6] This is the method used by Futagami et al Koichi Futagami, Yuichi Morita, and Akihisa Shibata, "Dynamic Analysis of an Endogenous Growth Model with Public Capital," *Scandinavian Journal of Economics* 95 (1993): 607.

$$z^{\cdot}/z = \dot{c}/c - k^{\cdot}/k = [(1-t_k)r - \rho] - [(1-t_w)w/k + (1-t_k)(1-\beta)\, g_P{}^{1-\beta} + (1-s)(\, t_w w + t_k rk)/k - z] \quad (14)$$

Create parameter F = βtk + (1- β)tw, such that

$$x^{\cdot}/x = sAFx^{1-\beta} - Ax^{2-\beta}(1 - sF) + z \quad (15)$$

$$z^{\cdot}/z = -Ax^{1-\beta}((1-s)F + (1 - \beta)(1 - tw)) \quad (16)$$

In equilibrium all variables will grow at the same optimal rate $x^{\cdot}/x = \dot{g}_I/g_I = k^{\cdot}/k = z^{\cdot}/z = \dot{c}/c$ so their relative proportions are constant. The balanced growth paths are found by setting $x^{\cdot}/x = z^{\cdot}/z = 0$ and solving for z. The resulting steady state equations are

$$z_{1*} = Ax^{*1-\beta}(1 - sF) + sFAx^{*-\beta} \quad (17)$$

$$z_{2*} = \rho + Ax^{*1-\beta}((1 - s)F + (1 - \beta)(1\text{-}tw)), \quad (18)$$

where * denotes the optimal value of a variable. These equations will define the stable balanced growth paths of proportions x and z. The equilibrium and comparative dynamics are well illustrated with a phase portrait of y against z. Figure 1 depicts the balanced growth paths of x and z. The solid lines denote the growth rates before the capital tax cut and the dotted lines denote those after the tax cut shock. The equilibrium growth rate for consumption, private capital and public capital is the intersection of these two lines.

Because of the nonlinearity of these equations, analytical solutions cannot be derived for the steady growth values of x and z. From this point forward the analysis proceeds numerically.

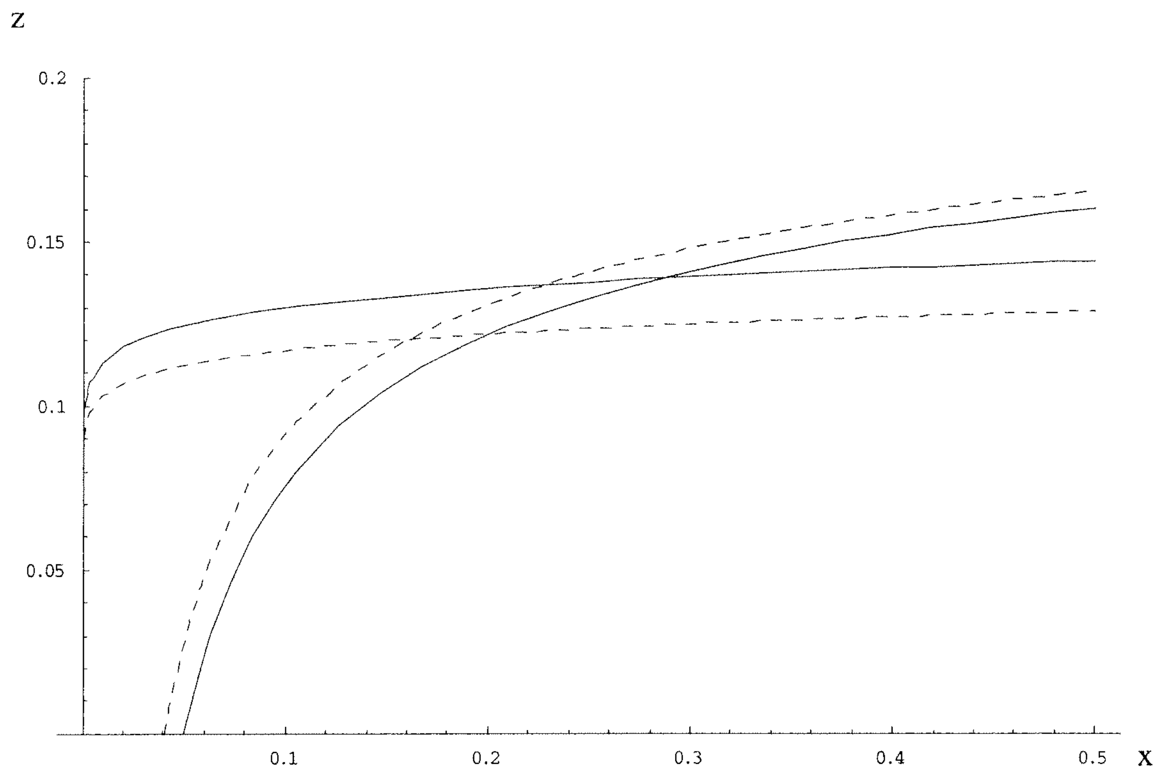

Figure 1. Balanced Growth Equilibriums Before and After a Capital Tax Cut

3.5 Finding Transition Paths

The transition path between the original and shifted balanced growth paths when the economy is shocked is now observed. The transition path is the route the economy follows from one steady state to the next. Since consumption is in direct, short-term control of the representative agent, it is able to change immediately with sudden tax changes. The stock of private capital, however, is not able to change instantaneously due to its illiquidity, and therefore takes longer to adjust. The decrease in taxes will initially decrease government expenditure, as to be expected, but the feedback effects set in as the tax base begins to grow, increasing tax revenue as the tax cut starts to "pay for itself".

The rate at which a shocked economy reaches its new equilibrium is derived from the convergence rates of the components, in this case k, g_I, and c. Before the economy was shocked, these three variables had been growing at the same equilibrium growth rate. A shock on the economy of a change in tax rates affects each variable differently, causing the economy to fall off of its balanced growth path. As discussed above with the transition path, consumption reacts immediately to a tax cut while public and private capital react slowly, due to their illiquid natures. Because of these differences in reaction time, each variable will reach the new steady state at a different rate. The variables also may move in a different direction in the long-run than they do initially. For example, if the government decreases capital taxes, t_k, the agent's resources are reallocated from consumption to capital accumulation, so short term consumption levels will drop. Investment in private capital accumulation will increase economic growth, creating more wealth for the agent, which it in turn can use for consumption at a higher level than before the tax cut, increasing long-term consumption to its new steady state. Similarly, government revenues, and hence public capital, will initially drop when a tax cut is put into effect since the smaller tax rate is being implemented on the same sized tax base. However, as the economy begins to grow and the tax base increases, g_I increases. Because g_I moves at a slow rate and, like consumption, changes signs as time goes by, its transition path is highly nonlinear.

Due to the nonlinear characteristics of the time paths of each variable, it is very difficult to find their transition paths. There are two widely used methods for finding transition paths: the Time Elimination Method and Taylor Series Expansions. Time Elimination is more accurate than a Taylor Series expansion when working with nonlinear functions, but must be analyzed numerically; it is not able to produce analytical equations for the convergence rates. Because of this, each new set of parameters will give different results for the convergence rates. Taylor Series expansions are able to produce closed-form solutions for the convergence coefficient, though it is at the expense of accuracy, since the Taylor Series expansion works by creating a linearized approximation of the nonlinear transition path equations. While the Taylor Series is accurate for economies that are close to their steady states, the farther an economy's starting point is from its equilibrium, the less accurate the results are.[7] Brief explanations of each method follow.

3.5.1 Time Elimination Method

Time Elimination begins by obtaining x as a function of z by replacing all x(t) variables with x(z), and all z(t) with z. The time variable, t, is now eliminated from the model since

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[7] Stinespring, John R. "Mathematica for Macroeconomics", Manuscript, 2007.

x(t) in any equation can be replaced by x(z). This is important since it allows any function containing x(t) to be made a function of z. Once the equations have been modified, time is added back into the model by changing all z to z(t). Using the new post-tax cut equation containing x as a function of z along with the initial condition of the equation for z at its original steady state, a system of differential equations may be formed to obtain an equation for z in terms of t, which is the time path of z. This is then used to find the time path of x.

As x and z as g_I/k and c/k, respectively, the time paths of x and z can be manipulated to find the time paths of g_I, k, and c. Again, this can only be done numerically, so any analysis is done by altering the parameter values and observing the effects.

3.5.2 Taylor Series Expansion

A Taylor Series Expansion finds a linear approximation to the nonlinear growth rates of x and z, which are then be used find their respective time paths.[8] Due to the nonlinearity of the model in this paper, the Taylor Series Expansion method is used to find transition paths.

A Taylor Series expansion uses matrix algebra to approximate growth rates. It begins by expanding the $x^{\cdot}$ and $z^{\cdot}$ equations and linearizing them around the steady state, or in this case the steady growth equilibrium[9]. This is done by partially differentiating each equation first by x, then by z. The partial differential of an equation with respect to x is multiplied by (x – xss) and added to the partial differential of the same equation with

[8] Stinespring, John R. "Mathematica for Macroeconomics", Manuscript, 2007.

[9] This procedure is used by Sims (1999), Novales and Ruiz (2002), and Novales et al. (1999).

respect to z, which is multiplied by (z – zss). The partial differentials linearize the equations while the multiplication of (x – xss) and (z – zss) assure that it is linearized around the steady state. The steady state values, xss and zss, are substituted into the partial differential equations for x and z. With the Taylor approximation, the accumulation rates of x and z become,

$$\dot{x} \approx x_{ss}(z - z_{ss}) + [(1-\beta)sFAx_{ss}^{-\beta} + z_{ss} - (2 - \beta)Ax_{ss}^{1-\beta}(1 - sF)](x - x_{ss})$$

$$\dot{z} \approx \{2z_{ss} - \rho - Ax_{ss}^{1-\beta}[(1 - s)F + (1 - \beta)(1 - t_w)]\}(z - z_{ss}) - (1-\beta)Ax_{ss}^{-\beta}[(1 - s)F + (1 - \beta)(1 - t_w)]z_{ss}(x - x_{ss}).$$

These two equations are now entered into a matrix and the eigenvalues and eigenvectors are calculated. The signs of the eigenvalues determine the type of transition path and the speed at which the economy reaches the new balanced growth equilibrium. The eigenvalues and eigenvectors form the equations for the dynamic time paths for x and z, which are then used to find the convergence rates of k and g_I.[10] The time path for the variables is given by,

$$x(t) = x_p + x_c = x_{ss} + v_{1,1}A_1e^{\upsilon 1t} + v_{1,2}A_2e^{\upsilon 2t}$$

$$z(t) = z_p + z_c = z_{ss} + v_{2,1}A_1e^{\upsilon 1t} + v_{2,2}A_2e^{\upsilon 2t},$$

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[10] Stinespring, John R. "Mathematica for Macroeconomics", Manuscript, 2007.

where υ_1 and υ_2 represent eigenvalues and $v_{i,1}$ and $v_{i,2}$, their corresponding eigenvectors. Saddle-path stability requires that there be one negative and one positive eigenvalue. Assigning the positive value to the first eigenvalue implies its eigenvector, $v_{i,1}$, will be zero and the solution system is rewritten as,

$$x(t) = x_p + x_c = x_{ss} + v_{1,2}A_2e^{\upsilon 2t}$$

$$z(t) = z_p + z_c = z_{ss} + v_{2,2}A_2e^{\upsilon 2t}.$$

For example, consider cutting t_k from 0.50 to 0.40 and assume A = .2, s = .1, tw = .25, β = 9/10, ρ = .05, γ = 1, δ = 0. The corresponding negative eigenvalue = -0.0469168 and its eigenvector consists of $v_{1,2}$ = 0.999506 and $v_{2,2}$ = 0.0314403. The steady states are x_{ss}^2 = 0.160537 and z_{ss}^2 = 0.120208.

$$x(t) = 0.160537 + 0.999506A_2e^{-0.0469168\,t}$$

$$z(t) = 0.120208 + 0.0314403A_2\,e^{-0.0469168\,t}$$

The initial condition we know is $x(0) = x_{ss}^1$ = 0.285151 while z(0) takes a discrete jump at t = 0 due to the consumption jump. Thus A_2 is (0.285151 - 0.160537)/0.999506 = 0.124676 and the dynamic paths are

$$z(t) = z_p + z_c = 0.120208 + (0.0314403)(0.124676)e^{-0.0469168t}$$

$$x(t) = x_p + x_c = 0.160537 + (0.999506)(0.124676)e^{-0.0469168t}.$$

These equations form the dynamic transition path. They are shown in Figure 2 as the linear transition path leading to the new steady state after the economy is shocked by a reduction of the capital tax rate from 0.50 to 0.40. It can be seen that the shock immediately causes the z ratio, c/k, to drop due to the substitution of investment in private capital accumulation for consumption. The jump lands on the transition path and slowly follows it as the stock of private capital increases to its intersection with the new balanced growth equilibrium.

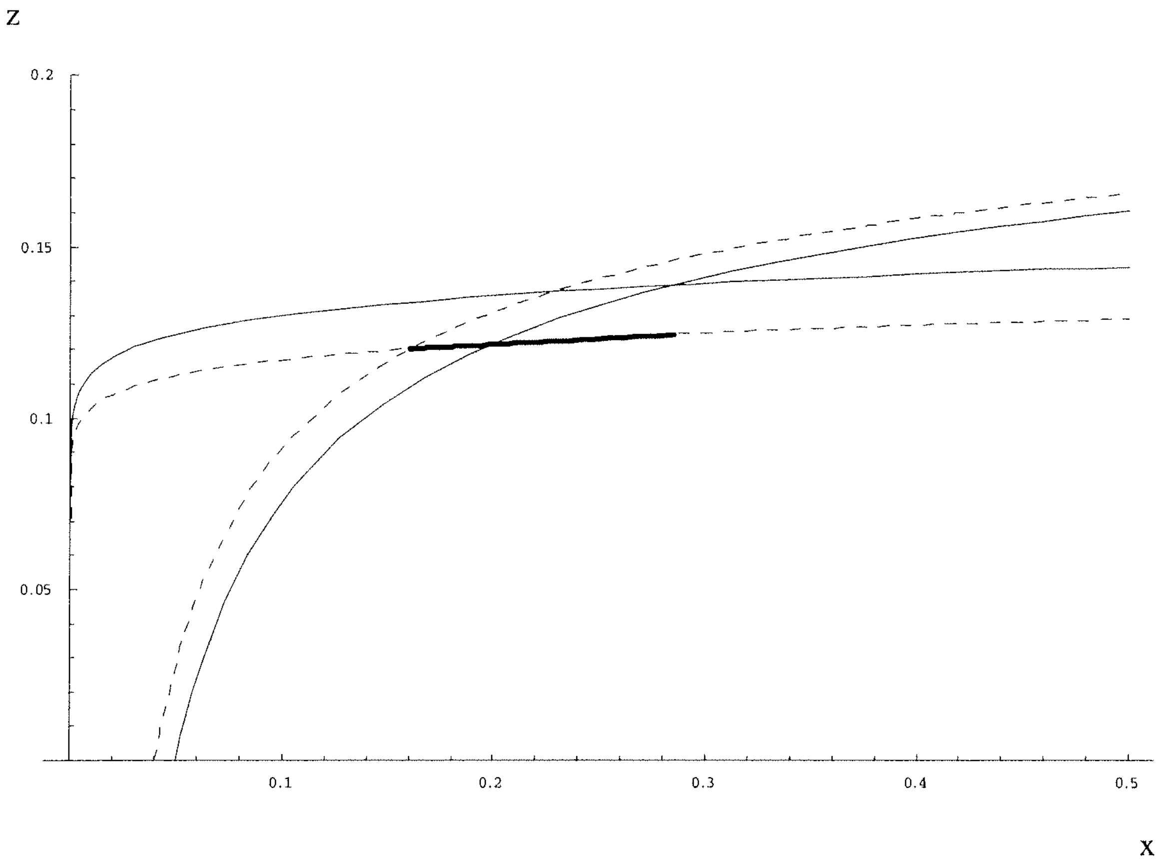

Figure 2. Transition path between steady states after a reduction in capital tax rates.

3.6 Feedback Effects

The ultimate goal of this paper is to see how a tax shock in the economy affects revenue growth. In a static situation, a tax cut would mean a proportionate cut in tax revenue, since growth effects are not taken into account; the economy is assumed not to alter working or spending behavior as a result of a reduced tax rate. A dynamic model takes these behavioral changes into account. While in a dynamic model a tax cut will immediately lower revenue, it allows for revenue to increase again as the tax base does. This post-cut increase in revenue due to an enlarged tax base is the feedback effect. The ability to measure feedback effects makes it possible to find an optimal tax rate reduction to decrease taxes while keeping tax revenue at a sufficient level for governmental needs.

As it is calculated in this paper, Dynamic Feedbacks = Growth Effects (Actual Revenues from one balanced growth path to the next) – Static Effects (new taxes multiplied by output growing at an unchanged rate). In an example of a capital tax cut from t_k^0 to t_k^1:

$$\text{Static Effects} = (t_k^1\beta + (1 - \beta)t_w^0)A[k(0)e^{\varphi 1t}]^{\beta}\,[g_I(0)e^{\varphi 1t}]^{1-\beta}$$

$$\text{Growth Effects} = (t_k^1\beta + (1 - \beta)t_w^0)A[k(t)]^{\beta}\,[g_I(t)]^{1-\beta}$$

To find the growth effects, k(t) and $g_I(t)$ are derived from x(t) and z(t) as follows. The values found for x(t) and z(t) can be plugged into the capital accumulation equations derived earlier in this paper:

$$\frac{\dot{g}_I}{g_I} = sFk^{\beta}g_I^{-\beta} = sF(\frac{1}{x(t)})^{\beta} \tag{1.1}$$

$$\frac{\dot{k}}{k} = y/k - c/k - g_I./k - \delta k/k - \delta g_I/k = x^{1-\beta} - z - (sFy - \delta g_I)/k - \delta - \delta x = x^{1-\beta} - z - sFx^{1-\beta} +$$

$$\delta x - \delta - \delta x$$

$$\frac{\dot{k}}{k} = (1 - sF)x(t)^{1-\beta} - z(t) \tag{1.2}$$

The dynamic path of revenues is found by substituting (1.2) and (1.1) into (1.0)

$$\frac{R.}{R} = \beta\frac{\dot{k}}{k} + (1 - \beta)\frac{\dot{g}_I}{g_I} \tag{1.0}$$

$$\frac{R.}{R} = \beta((1 - sF)x(t)^{1-\beta} - z(t) - \delta) + (1 - \beta)(sF(\frac{1}{x(t)})^{\beta} - \delta).$$

To solve for R(t), k_{ss}^1, k_{ss}^2, g_{ss}^1, g_{ss}^2 must first be found. Assume $k_{SS}^1 = 1$. With $x_{ss}^1 = g_{ss}^1$, $R_{ss}^1 =$

$$\frac{\dot{k}}{k} = (1 - t_w)\frac{w}{k} + (1 - t_k)r + \frac{(1 - s)(t_k r_I k + t_w w)}{k} - \frac{c}{k} - \delta$$

$$g_I./g_I = s(t_k r_I k + t_w w)/g_I - \delta$$

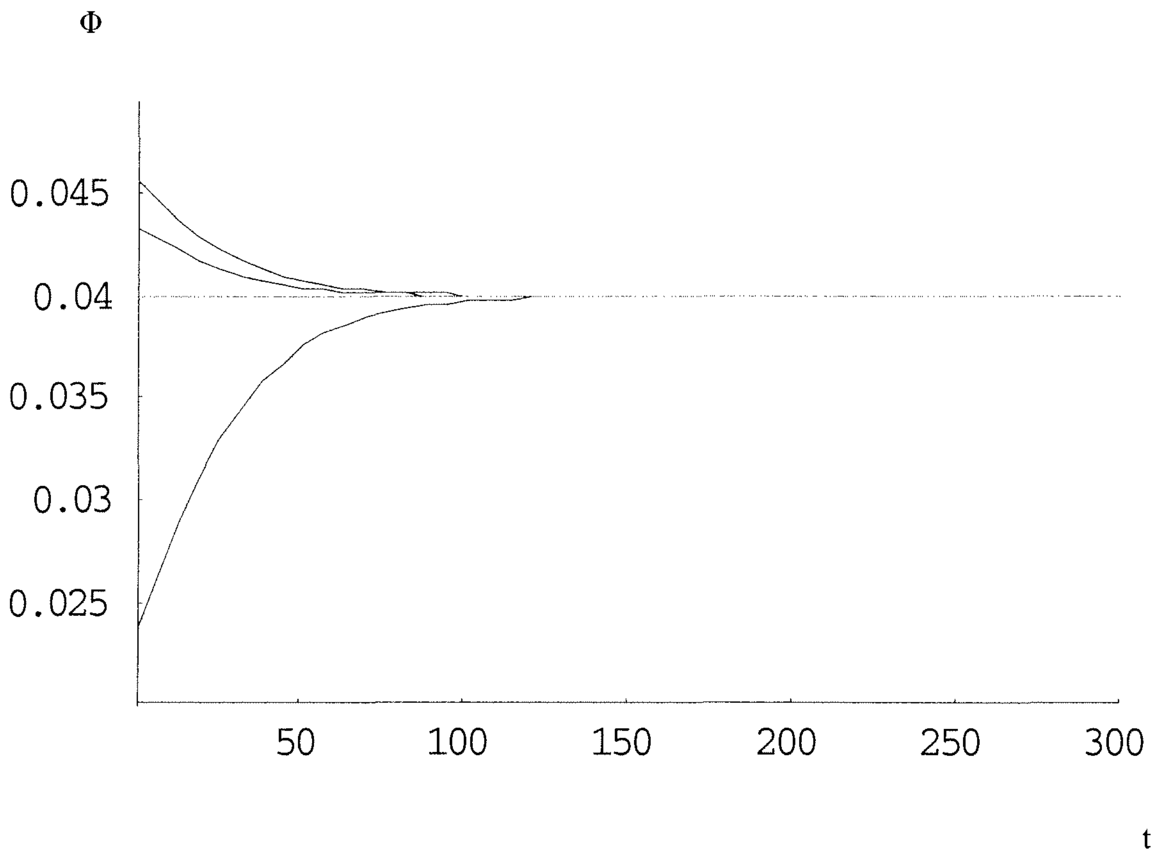

Figure 3. Convergence of k, g_I, and R at the post- tax cut balanced growth path.

Inserting the parameter values used in Section 3.5.2 into the dynamic paths for k, g_I, and R, a graph can be produced to show all three variables converging to the new steady state growth rate (Figure 3).

The feedback effect is measured as the present value of the additional tax revenues from the growth effect, discounted using the after-tax interest rate:

Feedback Effect = Present Value ($R_{dynamic} - R_{static}$).

The effects on revenue and growth of a permanent reduction in taxes are shown in Figures 4a and b with starting tax rates of $t_k = 0.50$ and $t_w = 0.25$. The horizontal axis shows the size of the tax cut. The vertical axis shows revenue change on the left and growth effect on the right. Revenue change measures the feedback effects as calculated above. The section of the graph where the revenue change is above zero indicates feasible tax cuts. At these points, the corresponding tax cut would be able to stimulate enough growth to increase the tax base to a point where it could compensate for the lost revenue from a permanent tax cut. The section of the graph where the revenue change is less than zero indicates infeasible tax cuts. The budget deficit resulting from the loss of revenue at that tax rate is too large to be compensated for in future growth without later fiscal adjustment.[11] The growth effect, shown on the right vertical axis, is measured as the change in the economic growth rate from its initial rate before the tax cut to its new steady state rate, calculated for each tax rate change. The growth rate change is also

[11] Alfonso Novales and Jesús Ruiz, "Dynamic Laffer Curves," *Journal of Economic Dynamics & Control* 27 (12 2002): 196.

graphed as an upward-sloping line in the capital tax scenario, and a downward-sloping line in the labor tax scenario.

The points where the revenue change lines intersect the x axis are the largest feasible tax cuts. In Figure 4a, the capital tax begins at .5, or 50%. In this scenario, the revenue change line intersects the x axis at roughly .16, indicating that the maximum feasible capital tax cut while holding labor taxes constant at 25% is .16, making the lowest feasible capital tax rate .34, or 34%; any rate smaller than this would violate the present value government budget constraint.[12] Novales and Ruiz find in their 2002 study that a higher initial tax rate will have a larger range of feasible tax reductions. Indeed, when the model in this paper is re-tested at tk1 = .65, the largest feasible tax cut is .31, or 31%. Figure 4b shows the revenue and growth effects of changes in the labor income tax rate, holding the capital tax rate constant at .5. The revenue change line never becomes positive, meaning any cut in labor income tax rates will permanently lower tax revenues and the economic growth rate. This is due to the fact that the labor supply in this model is inelastic. In an elastic labor supply situation, a reduction in the income tax rate would induce an increase in hours worked as people substituted labor for leisure. Increased hours worked earns a larger income, increasing the tax base. Even as the income tax rate is lower, the larger tax base could compensate for it, resulting in a range of feasible tax cuts. However, when the labor supply is inelastic, it is unable to respond to changes in the wage rate, so a decrease in income taxes will not increase the growth rate, eliminating the possibility of feedback effects from a larger tax base. In this case, the inability of the tax cut to spur growth leaves any change in the growth rate to be determined by other variables. Since output is a function of government expenditure, a decrease in g_l will not

[12] Ibid.: 196.

be offset by any larger effect, and will decrease output; a larger tax cut will only aggravate this negative effect. Decreasing output is analogous with a negative growth rate, shown in graph 4b.

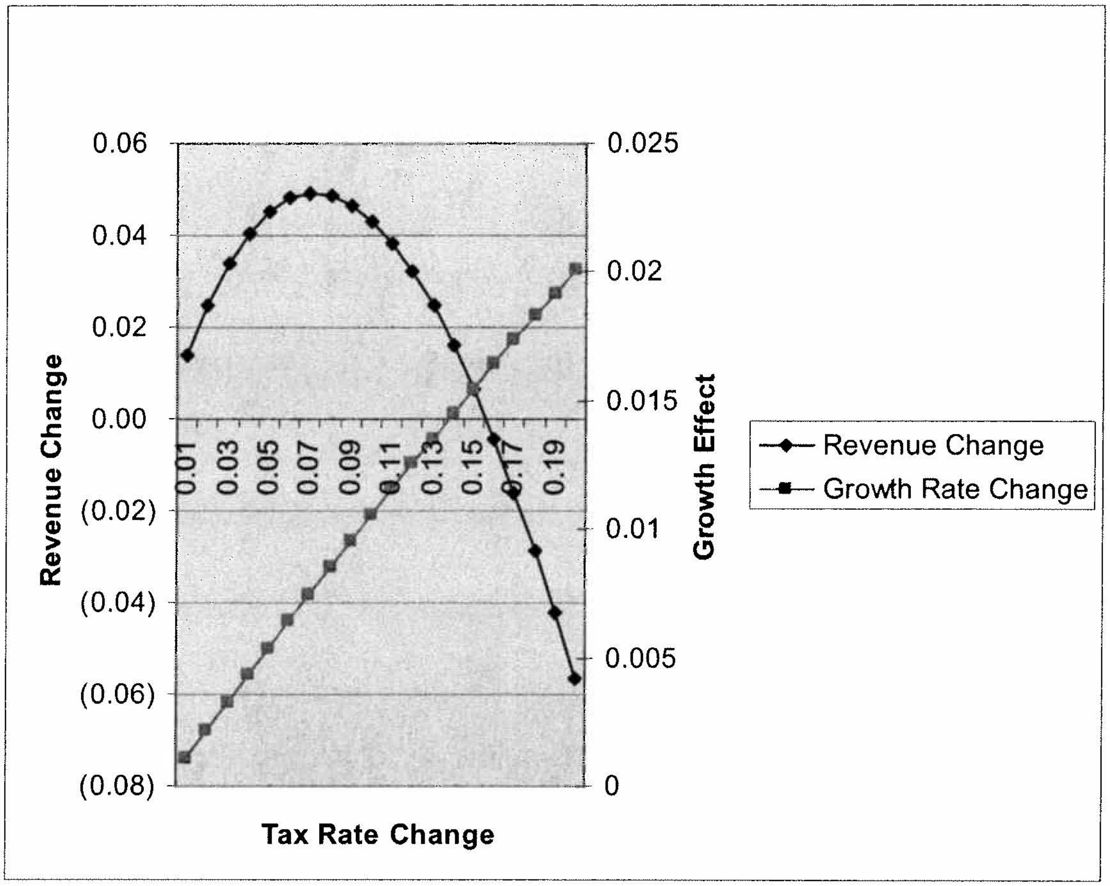

Figure 4a. Revenue and growth effects from capital tax rate cuts.

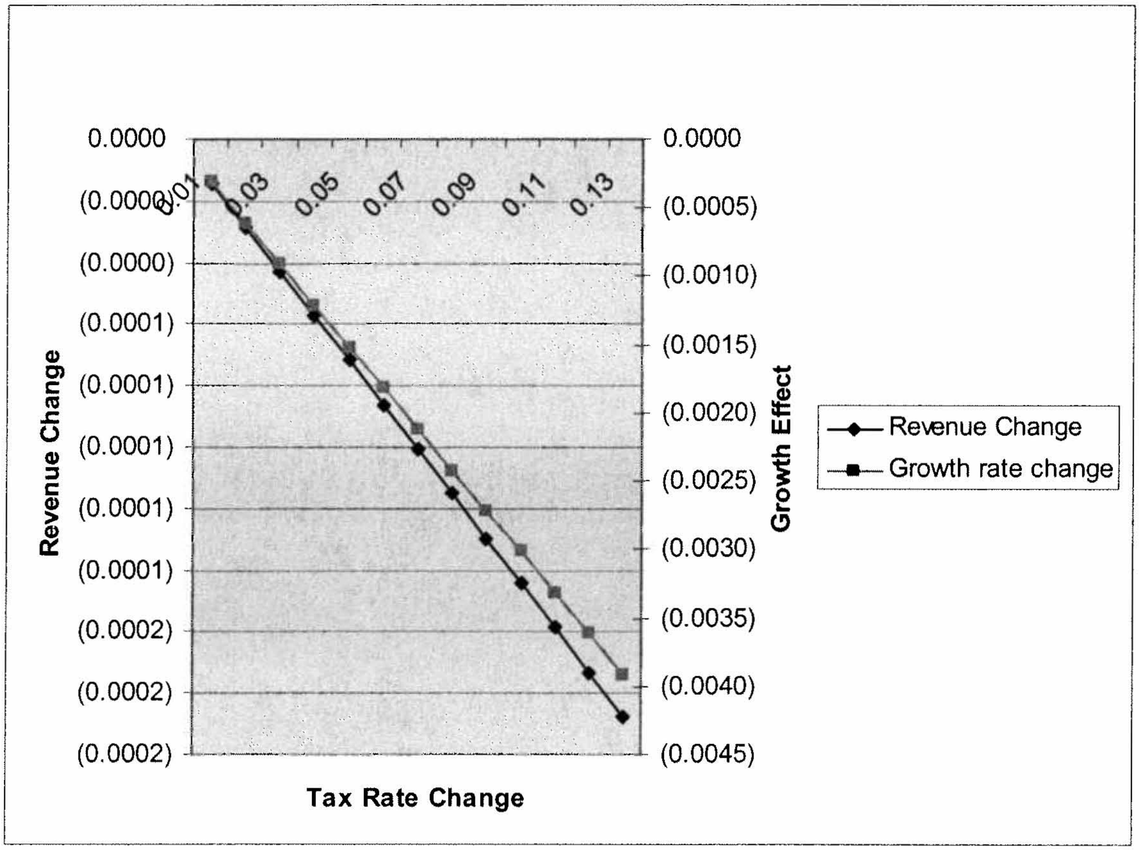

Figure 4b. Revenue and growth effects from income tax rate cuts.

The growth rate of revenue, R, over time is needed to find the feedback effects of the model. Assuming k(0) = 0 implies that $g_I(0) = x^*1$. Using these values, the time paths of k and g_I are calculated from the equations derived at the beginning of this chapter. Inserting these into the revenue function,

$$R_{dynamic} = FA\, k^{\cdot\beta} \dot{g}_I^{1-\beta}.$$

Calculating the static time path of revenue gives,

$$R_{static} = (tk\beta + (1-\beta)tw)A((k_{ss}^{1})^{\varphi t\beta})\, ((g_{ss}^{1})^{\varphi t(\beta-1)}).$$

As stated above, the feedback effect is the present value of the additional tax revenue earned in a dynamic situation:

$$Feedback = PV\,(R_{static} - R_{dynamic}).$$

For the parameter values used above, the feedback effect from a capital tax cut from 50% to 40% is 4.31%. The growth effect is 1.06%.

CHAPTER 4

LAFFER CURVE ANALYSIS AND COMPARATIVE STATICS

In this chapter, different ways to assess the meaning and significance of this model by analyzing its various components are examined. The first part of this section will illustrate the feedback effects with Laffer curves, explaining the background and meaning of the Laffer curve. The next part will use comparative statics to test the model.

4.1 Laffer Curve Analysis

As explained above, the Laffer curve illustrates revenue earned at different tax rates. The three significant points on the graph are a) the two x-axis intercepts, where tax rates equal 0% and 100%, and b) the peak of the curve. At a 0% tax rate, the government will not gain any revenue. At a tax rate of 100%, the population has no incentive to work since they will not be able to keep any of their earnings, so again tax revenue is zero. The peak of the curve is the tax rate which will garner the largest possible government revenue[1]. To the left of the curve, an increase in tax rates will increase revenue. However, on the right side of the peak, it is a *decrease* in tax rates that will raise revenue, and the tax cut will pay for itself. Dynamic scoring does not seek the tax rate that will earn the most revenue, but rather the point on the curve representing the minimum tax rate necessary to support

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[1] Arthur B. Laffer and R. David Ranson, "A Formal Model of the Economy," *Journal of Business* 44 (07 1971): 247-270.

the constant exogenous stream of government expenditures[2]. Feasible tax cuts are those that will, through growth, create sufficient revenue to finance government expenditures. Given the range of feasible tax cuts for a specific initial tax rate, dynamic scoring aims to find the largest of these since it is the largest feasible cut that will give the smallest feasible tax rate. The optimal size tax cut calculated for this model in Chapter 3.6 is .16, giving a lowest feasible capital tax rate of 34%.

4.2 Comparative Statics

Isolating variables and parameters and observing how changes in them affect the rest of the model yields interesting information on the relative significance of the model's various components and is the basis of comparative statics. First analyzed is beta, the output elasticity of private capital and indirectly of public capital. The value for beta used in the model in Chapter 3 is 9/10. As the output elasticity of private capital, beta measures the sensitivity of output to changes in private capital stock; the output elasticity of public capital stock is 1/10. If beta is decreased from 9/10 to 7/10, the effects of changes in private capital stock on output are much less pronounced. A decrease from the capital tax rate of 50% to 40% increases growth by 1.06% when beta is 9/10. When beta is decreased to 7/10, the growth rate increases by 0.209%. This change is due to beta's role in measuring the sensitivity of output to changes in private capital levels. A larger beta signifies a higher sensitivity to changes in private capital stock, so it is fitting that when output is more sensitive to changes in k (when beta is larger), it will react more

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[2] Alfonso Novales and Jesús Ruiz, "Dynamic Laffer Curves," *Journal of Economic Dynamics & Control* 27 (12 2002): 181.

strongly to changes in the level of private capital from changes in tax cuts than it would were its sensitivity, beta, lower.

Another variable to be examined in the model is rho. Rho, ρ, measures consumer time preference: the willingness of the consumer to forego consumption today and invest in private capital which will increase wealth, making the consumer able to consume a larger amount in the future. Rho in the initial model is .05. When capital taxes drop from 50% to 40%, the growth rate increases by 1.06%. If rho is changed to .03, the consumer is more likely to substitute future consumption for present consumption, and the growth rate increases by 1.09%. A higher rho signifies a less patient consumer, whereas a lower rho is indicative of a more patient consumer who will more willingly substitute future consumption for present consumption. A consumer with a lower rho is more reactive to a change in tax rates, so a tax cut for a consumer with a low rho will be much more effective and more likely to pay for itself.

The government savings rate, s, measures the proportion of tax revenue that will be used for government investment versus government transfers. S in this model is .1, meaning that 10% of tax revenue is used for investment in public capital. At this rate, the capital tax cut shock increases the growth rate by 1.06%. If s is increased to .3, the shock increases the growth rate by 1.19%. A savings rate of .3 means that the government is saving 30% of its revenue for investment in public capital. Since public capital not only is a factor in output, but also enhances the productivity of private capital[3], it makes sense that a larger government savings rate will increase growth at a higher rate.

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[3] David A. Aschauer, *Is Public Expenditure Productive? Manuscript,* (Chicago: Federal Reserve Bank Chicago, 1988)

CHAPTER 5

CONCLUSION

In this paper, I examined the effects of tax cuts on tax revenue, a topic that has been the focus of political debates since the 2001 implementation of the Bush Economic Growth and Tax Reconciliation Act, and the 2003 Jobs Growth and Tax Relief Reconciliation Act. Previous analyses have not taken into account the growth effects from tax cuts, observing only the short-term effects. Since a cut in tax rates will decrease tax revenue proportionally in the first period after the cut, "static scoring" will always project an unrecoverable revenue loss. A new method of measuring these revenue effects has generated significantly different results by analyzing the tax cuts in a long-term "dynamic scoring" model. Dynamic scoring observes the changes in the economy in the longer-term as is migrates from its original pre-cut equilibrium to its new post-cut equilibrium. The model created here endogenizes the growth variable in a dynamic scoring model with public capital. Endogenous economic growth is more realistic than exogenous growth since in the real world the growth rate is dependent on changes in other variables such as tax rates, private capital accumulation rates, savings rates, and myriad more. The inclusion of public capital in this model affected the growth rate by augmenting the effects of capital tax cuts on the economy. This was due to the assumption of productive government spending, meaning that public capital increases the productivity of private capital, so an increase in the public capital stock increases the productivity of the existing private capital stock and hence the growth rate increases.[1]

In this paper, capital taxes are cut, inducing a higher rate of private capital accumulation. This in turn increases income which increases the tax base. A smaller percentage tax cut implemented on a larger tax base will produce tax revenues higher than the immediate-run assumption that tax revenue will decrease in proportion to the tax cut. This extra revenue gained from the enlarged tax base is called the "feedback effect". Dynamic scoring argues that this feedback effect is essential in predicting the effects of a tax cut as it produces much different results on the change in size of the post-cut tax revenue.

The typical article on dynamic scoring models researched for this paper uses the growth rate of the economy as an exogenous variable. Therefore, this research is unique in having the growth rate be endogenous. Of the literature reviewed on dynamic scoring and endogenous growth, the study that came closest to mine was the 1996 paper by Novales and Ruiz[2] and my results are consistent with theirs. The Novales and Ruiz paper used endogenous growth, but not in a dynamic scoring model. Their study was of endogenous growth in a Laffer curve model. While these are similar in the use of endogenous growth, dynamic scoring models are not in the same family as dynamic Laffer models. This paper does not focus on income taxes, only capital taxes. The results show that after the feedback effects, a decrease in capital tax rates will more than make up for the initial reduction in tax revenue.

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[1] David A. Aschauer, *Is Public Expenditure Productive? Manuscript,* (Chicago: Federal Reserve Bank Chicago, 1988)

[2] Alfonso Novales and Jesús Ruiz, "Dynamic Laffer Curves," *Journal of Economic Dynamics & Control* 27 (12 2002): 181.

The results of this paper are extremely relevant today amidst the Bush tax act debates. The existence of feasible tax cuts is a strong argument against letting the EGTRRA and JGTRRA tax cuts expire at their intended dates. The scope of this paper consisted of devising an endogenous growth model of dynamic scoring with public capital and an inelastic labor supply. Calibration of the model to the United States economy was not within the focus of this study, and therefore would be a direction for further research. Calibrating this model to the United States and analyzing tax scenarios currently being discussed in the political realm would produce very interesting and relevant results. An elastic labor supply would also be a topic for further study. Since the model in this paper used a completely inelastic labor supply, the effects of income tax cuts were not able to be discerned. The working paper "Dynamic Laffer Curves, Economic Growth and Public Capital"[3] will do exactly that, expanding upon the model in this paper by adding a labor-leisure choice and testing the economic ramifications of cuts in the labor income tax rate.

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[3] John R. Stinespring, "Dynamic Laffer Curves, Economic Growth and Public Capital," *Working Paper* (2007)

Appendix A

Guide to Variables

k	Private capital per capita
g_I	Public capital per capita
y	GDP per capita
c	Consumption per capita
A	Technology constant
τ	Flat tax rate
β	Output elasticity of k, g_I, c
x	g_I/k
z	c/k
r	Rental rate of k
w	Wage rate
g_T	Government transfers to households
h	Hours worked
i_G	Government investment in public capital per capita
tk	Capital tax rate
tw	Income tax rate
s	Government savings rate
δ	Depreciation
γ	Inverse of intertemporal elasticity of substitution
ρ	Consumer time preference
i_K	Investment in private capital per capita
R	Tax revenue
Φ	Growth rate
λ	Shadow price of k, g_I, c
υ_1, υ_2	Eigenvalues
ν_1, ν_2	Eigenvectors
φ	Constant growth rate of output

Appendix B

Revenue and Growth Change From Capital Tax Cuts

t_k	$\Delta\varphi$	ΔRevenues
0.5-0.49	0.00108961	0.0137428
0.5-0.48	0.00217285	0.0249692
0.5-0.47	0.00324928	0.033864
0.5-0.46	0.00431847	0.0405953
0.5-0.45	0.00538001	0.0453142
0.5-0.44	0.0064335	0.0481573
0.5-0.43	0.00747853	0.0492482
0.5-0.42	0.0085147	0.048699
0.5-0.41	0.00954159	0.0466113
0.5-0.40	0.0105588	0.0430761
0.5-0.39	0.0115659	0.0381817
0.5-0.38	0.0125625	0.0320004
0.5-0.37	0.013548	0.0246034
0.5-0.36	0.0145221	0.0160545
0.5-0.35	0.0154842	0.00641201
0.5-0.34	0.0164338	-0.00427083
0.5-0.33	0.0173704	-0.0159452
0.5-0.32	0.0182933	-0.0285664
0.5-0.31	0.019202	-0.0420933
0.5-0.30	0.0200957	-0.0564884

Appendix C

Revenue and Growth Change From Income Tax Cuts

t_w	$\Delta\varphi$	ΔRevenues
0.25-0.24	-0.0000142962	-0.00030438
0.25-0.23	-0.000028619	-0.000608063
0.25-0.22	-0.0000429685	-0.000911169
0.25-0.21	-0.0000573448	-0.0012137
0.25-0.20	-0.000071748	-0.00151565
0.25-0.19	-0.0000861782	-0.00181703
0.25-0.18	-0.000100636	-0.00211478
0.25-0.17	-0.00011512	-0.00241808
0.25-0.16	-0.000129632	-0.00271772
0.25-0.15	-0.000144172	-0.00301679
0.25-0.14	-0.000158739	-0.00331527
0.25-0.13	-0.000173333	-0.00361318
0.25-0.12	-0.000187956	-0.0039105

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