

MODELING ATTITUDES TOWARDS MATHEMATICS IN DENVER PUBLIC HIGH
SCHOOLS USING STRUCTURAL EQUATION MODELING

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MODELING ATTITUDES TOWARDS MATHEMATICS IN DENVER PUBLIC HIGH SCHOOLS USING PARTIAL LEAST SQUARES STRUCTURAL EQUATION MODELING

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ABSTRACT. This paper will undertake an investigation into attitudes towards mathematics. Data from 27 Denver Public High Schools¹ reports that all students in the DPS high schools score on average 15% higher on reading/writing standardized tests than on math. The study speculates that this stems from attitudinal differences towards the subjects that students have. This study proposes a mathematical model that hypothesizes the school's environmental make-up, students home environment, and students performance on standardized tests affect students attitudes toward mathematics. The model will be implemented in the software package SMART-PLS that uses a partial least squares regression algorithm to estimate the latent variable "Attitudes Towards Mathematics" and investigate what factors affect math education. The study shows that *Attitudes Towards Math* affect performance on *Standardized Testing* and *Home Environment* affects students *Attitudes Towards Math* and *Standardized Testing Performance*.

1. Introduction: Structural Equation Modeling

Structural Equation Modeling (SEM) is a multivariate data analysis method. The SEM-based technique has stemmed from and is closely related to techniques such as principal component analysis, factor analysis, and multiple regression. The social sciences have found SEM particularly useful as many of its studies aim to investigate variables that are not directly measurable. As with an attitude towards math, many real-world phenomena are not as blatantly causal as they seem but are the effect of secondary factors that are difficult to directly link. The SEM-based technique allows for an adjustable model based in theory and hypothesis to be tested and analyzed with data using path and factor analysis, covariance algebras, and various statistical tests.

1.1. **Overview of SEM.** To look at SEM, the following terms are helpful,

- Measured Variable: Observed variables/indicators, predictors and outcomes in path analysis. Also known as independent.
- Latent Variable: Un-observable variable, the driving constructed variable for measured variables. Also known as dependent.
- Formative Indicator: The formative measurement scale exists when indicators cause the latent variable and cannot be interchanged amongst one another.
- Reflective Indicator: The reflective measurement scale exists when the indicators are correlated and interchangeable.
- Exogenous Variable: A variable that is not caused by another variable.

- Endogenous Variable: A variable that is caused by one or more other variables. Endogenous variables may cause other endogenous variables.
- Error: Variance remaining after prediction of a measured variable.
- Disturbance: Variance remaining after prediction of latent variable.
- Direct Effect: Regression coefficients of direct prediction.
- Indirect Effect: Mediating effect of exogenous variable (x_1) on endogenous variable (y) through another exogenous variable (x_2).
- Covariance Structure: Relationships based on variance and covariance.

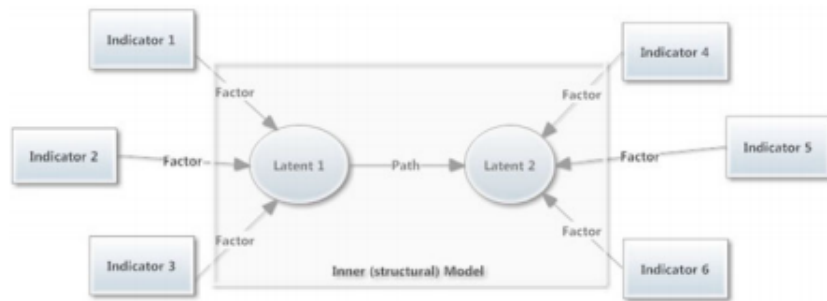
2. Steps of SEM

Step 1 Model Specification: The first step in SEM is to specify a model based on theory and research. The model generally has two sub-models: the inner model, sometimes referred to as a structural model, and the outer model, sometimes referred to as a measurement model. These sub-models are connected by single or double headed arrows called paths. A single headed arrow denotes a prediction that one variable has an effect on another variable. A double headed arrow denotes a correlation. If there are no arrows between variables, this denotes a hypothesized absence of relationship. Each path will have an associated parameter, some of which will be estimated and some fixed.

Outer Models The outer model examines the relationships between the measured variables and the latent variables. The relationships are indicated by the estimated path coefficients, associated errors, and standard errors. This information gives an indication of the fit of the model.

Inner Models: The inner model examines the relationships between the latent variables. The relationships are indicated by the estimated path coefficients, associated disturbances, and standard errors. The model is constructed to assess how the latent variables relationships behave and the fit of the model.

FIGURE 1. Generic Latent Variable Path Model



Step 2 Identification: A model can be *under-identified*, meaning there are more unknown parameters than known parameter and not all parameters can be estimated. The model can be *just-identified*, meaning there are the same number of known and unknown parameters, or the model can be *over-identified*, meaning there are more known parameters than unknown parameters. Essentially, this step identifies the degrees of freedom in the model.

Step 3 Model Estimation: Model estimation estimates all parameters in the

model. This step mainly occurs in computer programs. SMART-PLS, the program used in this study, utilizes a partial least squares regression algorithm to estimate the parameters.

Step 4 Model Fit: The estimated model parameters are used check the model fit. To evaluate model fit, one should examine standardized residuals, R^2 values for endogenous variables, t statistics, and inner and outer path coefficients. "Overall model fit obviously should be assessed by considering both statistical fit and model interpretability" (Kunnan 308).

Step 5 Model Respecification: When a model has not demonstrated good model fit, the models' parameters are respecified and then re-estimated. This process continues until the model has good fit.

2.1. Underlying Parameters in SEM. Now, we will explore the underlying mathematics of SEM. It has been mentioned that SEM is based in regression and covariance algebras, so it follows that the underlying parameters in SEM would be regression coefficients, variances, and covariances. To recall some covariance algebra rules, the equation for expected value for discrete random variables is

$$E[x] = \sum_{i=1}^N x_i p_i$$

where x_i is an outcome and p_i is the probability of the outcome and the equation for covariance is

$$\begin{aligned} COV(X_1, X_2) &= E[(X_1 - E[X_1])(X_2 - E[X_2])] = \\ &E[X_1 X_2] - E[X_1]E[X_2]. \end{aligned}$$

Some important rules to remember specifically for SEM are

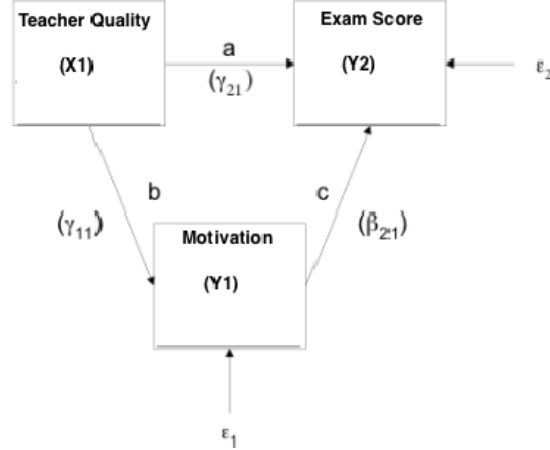
$$\begin{aligned} COV(c, X_1) &= 0 \\ COV(cX_1, X_2) &= c \cdot COV(X_1, X_2) \\ COV(X_1 + X_2, X_3) &= COV(X_1, X_3) + COV(X_2, X_3), \end{aligned}$$

where c is a constant. And lastly, we should recall that a regular basic regression is

$$Y = bX + a$$

Now, we can take a look at a simple example model.

FIGURE 2. Example Latent Variable Path Model



Given this simple latent variable model, these are the equations.

$$Y_1 = \gamma_{11}X_1 + \epsilon_1$$

Y_1 is predicted by X_1 and ϵ_1 . X_1 is an exogenous variable so γ is used to indicate the weight.

$$Y_2 = \beta_{21}Y_1 + \gamma_{21}X_1 + \epsilon_2$$

Y_2 is predicted by X_1 and Y_1 and error. Y_1 is endogenous thus the weight is indicated by β . The different weights are used to indicate the relationships between the type of variables the path connects. So, γ weights indicate a relationship between a exogenous variable and an endogenous variable while β weights indicate a relationship between two endogenous variables.

In this model there is an example of an estimated covariance with a simple path and a complex path. The simple path is between the exogenous and endogenous variables, $COV(X_1, Y_1)$. We know $Y_1 = \gamma_{11}X_1 + \epsilon_1$.

$$COV(X_1, Y_1) = COV(X_1, \gamma_{11}X_1 + \epsilon_1)$$

now recall equations covariance rules above,

$$\begin{aligned} COV(X_1, \gamma_{11}X_1 + \epsilon_1) &= COV(X_1, \gamma_{11}X_1) + COV(X_1, \epsilon_1) \\ COV(X_1, \gamma_{11}X_1) &= \gamma_{11}COV(X_1, X_1) \end{aligned}$$

where we know $COV(X_1, X_1)$ is the same as the variance of X_1 . So,

$$COV(X_1, Y_1) = \gamma_{11}\delta_{X_1, X_1}$$

The complex path is between two endogenous variables, $COV(Y_1, Y_2)$. Substituting the equations for Y_1 and Y_2 and then distributing the pieces,

$$\begin{aligned} COV(Y_1, Y_2) &= COV(\gamma_{11}X_1 + \epsilon_1, \beta_{21}Y_1 + \gamma_{21}X_1 + \epsilon_2) \\ COV(Y_1, Y_2) &= COV(\gamma_{11}X_1, \beta_{21}Y_1) + COV(\gamma_{11}X_1, \gamma_{21}X_1) + \\ &COV(\gamma_{11}X_1, \epsilon_2) + COV(\epsilon_1, \beta_{21}Y_1) + COV(\epsilon_1, \gamma_{21}X_1) + COV(\epsilon_1, \epsilon_2) \end{aligned}$$

Nothing in the equation correlates with ϵ so,

$$COV(Y_1, Y_2) = COV(\gamma_{11}X_1, \beta_{21}Y_1) + COV(\gamma_{11}X_1, \gamma_{21}X_1)$$

any by rearranging,

$$COV(Y_1, Y_2) = \gamma_{11}\beta_{21}(X_1, Y_1) + \gamma_{11}\gamma_{21}COV(X_1, X_1)$$

$$COV(Y_1, Y_2) = \gamma_{11}\beta_{21}\delta_{x_1y_1} + \gamma_{11}\gamma_{21}\delta_{x_1x_1}$$

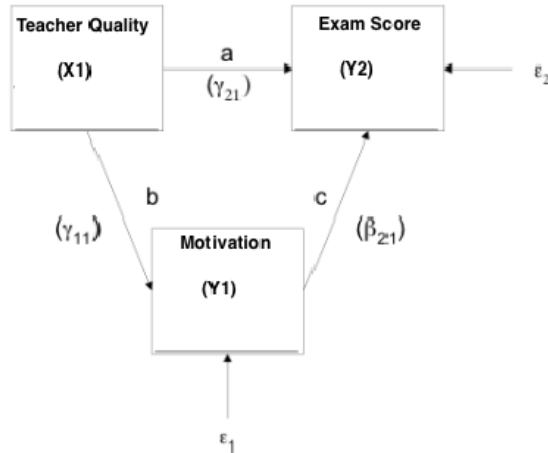
Using the above equations, $COV(X_1, Y_2)$ is very easy.

$$COV(X_1, Y_2) = COV(x_1, \beta_{21}Y_1 + \gamma_{21}X_1 + \epsilon_2) =$$

$$COV(X_1, Y_2) = \beta_{21}\gamma_{11}\delta_{x_1x_1} + \gamma_{21}\delta_{x_1x_1}$$

Referring back to the model,

FIGURE 3. Example Latent Variable Path Model



$$COV(X_1, Y_1) = \gamma_{11}\delta_{X_1, X_1}$$

$$COV(Y_1, Y_2) = \gamma_{11}\beta_{21}\delta_{x_1y_1} + \gamma_{11}\gamma_{21}\delta_{x_1x_1}$$

$$COV(X_1, Y_2) = \beta_{21}\gamma_{11}\delta_{x_1x_1} + \gamma_{21}\delta_{x_1x_1}$$

We can see how these equations are taken from the relationships in the model. Using these equations, a covariance matrix is built like so,

TABLE 1. Sample Covariance Matrix

	X1	Y1	Y2
X1	var(X1)	cov(X1,Y1)	cov(X1,Y2)
Y1	cov(Y1,X1)	var(Y1)	cov(Y1,Y2)
Y2	cov(Y2,X1)	cov(Y2,Y1)	var(Y2)

The covariance between two variables is the correlation multiplied by the product of the variables' standard deviations. The covariance of a variable with itself is the variable's variance. The covariance matrix shows these values in correlation with the model. The goal is to construct a model with a implied covariance matrix that is not significantly different from the sample covariance matrix. The sample covariance matrix is constructed using the initial values of the un-estimated model, and the implied covariance matrix is constructed using the values of the estimated model. This is one test SEM utilizes to evaluate the goodness of fit of the model.

3. Partial Least Squares Path Modeling

Partial Least Squares (PLS) path modeling was developed as a computer implementation for econometrics². PLS is not the conventional structural equation modeling technique; however for some research objectives it is more suitable. PLS is often the least restrictive of the multivariate data analysis methods, and for this reason was appealing to this study. PLS is used for theory confirmation, to find where multivariate relationships between certain variables exist, and to analyze such relationships.

The **PLS algorithm** used in SMART-PLS occurs in three steps.

1) The algorithm runs an iterative estimation process of the latent variable scores which repeats until convergence occurs or the maximum number of iterations have occurred. The iterative process goes as follows:

- a) outer approximation of the latent variable scores
- b) estimation of the inner weights
- c) inner approximation of the latent variable scores
- d) estimation of the outer weights.

2) Then the algorithm estimates the outer weights/loading and path coefficients.

3) Finally the algorithm estimates the location parameters.

Explaining the Algorithm

1) **(a)** First, the data for all the cases for each variable is normalized. Then outer path weights are initialized, usually at 1. Then the initial outer path weights are multiplied by the normalized value of each indicator for each case. These values are added together for each latent variable. Then this value is normalized to give *outer approximation of latent variable scores*.

(b) The *inner path weights* are estimated next by taking the covariance of the latent variable scores from (a) of any two latent variables there is a path between.

(c) To find the *inner approximation of latent variable scores* for exogenous latent variables, the path coefficient (b) is multiplied by the normalized outer approximation value (a) of the endogenous variable. For endogenous latent variables, the outer approximation value (a) of its exogenous latent variable is multiplied with its respective path coefficient. This occurs for each exogenous variable that causes the endogenous variable and these values are added together.

(d) Finally, the outer weights are re-estimated. To find the *estimation of outer weights*, the inner estimations (c) are multiplied by the outer initial weights and then normalized.

To run the next iteration, the outer path weights would be initialized at the re-estimated values i.e. step (a). Then steps (b)-(d) would run again. This process

continues until the values of the indicator outer weights have converged.

2) Next, the outer loadings are calculated by taking the covariance of the normalized input data with the data from the final outer estimation. The path coefficients are estimated by a linear regression specified by the program.

3) The location parameters are similarly calculated by a linear regression specified in the program.

3.1. Weighting Scheme. The model weighting options offered by SMART-PLS are

- centroid weighting scheme
- factor weighting scheme
- path weighting scheme

These different weighting scheme offer slightly different results however using any one of them is ideal. The use of the schemes allows for obtaining the highest R^2 value for endogenous latent variables. This study utilized the path weighting scheme which is the default setting for SMART-PLS.

4. Motivation

The Denver Public School data shows that for all 27 high schools in the study, standardized reading and writing test scores are higher than math.

- 70% of all students not eligible for free or reduced lunch (FRL) in 2013 were proficient and above on the Math CSAP/TCAP.
- 37% of all students eligible for free or reduced lunch (FRL) in 2013 were proficient and above on the Math CSAP/TCAP.
- 45% of all students in 2013 were proficient and above on the Math CSAP/TCAP.
- In comparison, 60% of all students were proficient and above on the English/Reading CSAP/TCAP. That's a 15% difference.

This study speculates that reading and writing are more fully integrated into childhood from a young age while mathematics is left out. The purpose of this study is to look into the underlying relationships of various variables and attitudes towards math. These variables are observed to be causally linked to how students feel about math. This study looks to mathematically investigate these observations to help the Denver Public School system target factors that are most affecting students attitudes towards and interest in math. The study is also interested in the effects of integration on attitudes towards learning.

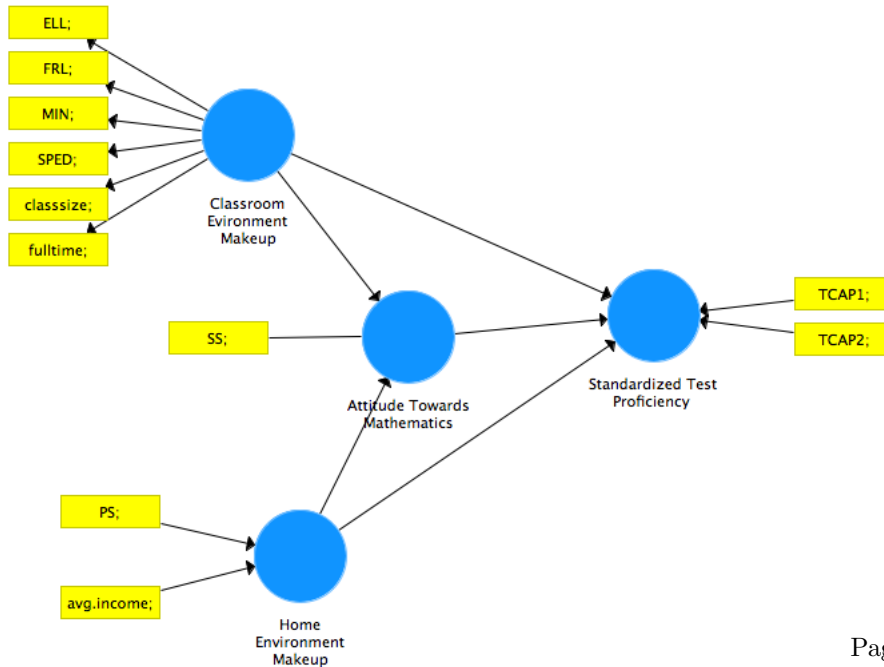
5. Specifying the Attitudes Towards Math Model

The attitudinal model to examine attitudes towards math in Denver Public High Schools looks 4 latent variables with 10 different indicators. The indicators and other latent variables demonstrate what this study believes to be the underlying causes of an "attitude". The indicator data make up is shown in Table 2 and the outer and inner model structure is shown in Figure 4.

TABLE 2. Indicator Data

Indicator	Data
School Environmental Makeup	
class size_	# of students in class
fulltime_	# of full time teachers employed
frl_	% eligible for free and reduced lunch
ell_	% english language learners
min_	% minority
sped_	% special education
Home Environment Makeup	
ps_	SPF score card rating of parental encouragement and satisfaction
avg.income_	average household income
Attitudes Towards Math	
ss_	SPF score card rating of student engagement and satisfaction
Standardized Testing Proficiency	
TCAP1_	% proficient in math
TCAP2_	% proficient in reading/writing

FIGURE 4. Attitudinal Model



5.1. Information from SMART-PLS in Attitudinal Model. After running the consistent PLS algorithm the first time the following results were found. In this section the results of the algorithm will be explained.

(i) Explanation of target endogenous variable variance

- The coefficient of determination, R^2 is 0.542 for the target latent variable Attitudes Towards Mathematics. This means that the two latent variables, Class Environment and Home Environment moderately³ explain the 54.2% variance in Attitudes Towards Mathematics.
- Together the three latent variables, Attitudes Towards Math, Class Environment, and Home Environment, explain 70.6% of the variance of Standardized Testing Proficiency.

TABLE 3. Inner Model

	Attitudes Towards Math	Classroom Environment	Home Environment	Standardized Testing Proficiency
Attitudes Towards Math				0.967
Classroom Environment	-0.181			-0.240
Home Environment	0.658			-0.419
Standardized Testing Proficiency				

(ii) Inner model path coefficient sizes and significance

- The inner model suggests that Attitudes Towards Math has the strongest (0.967) effect on Standardized Testing followed by Classroom Environment (-0.240) and then Home Environment (-0.419).
- Home Environment has the strongest (0.658) effect on Attitudes Towards Math followed by Class Environment (-0.181).
- The hypothesized path between Attitudes Towards Math and Standardized testing and the hypothesized path between Home Environment and Attitudes Towards Math are statistically significant.
- However, the hypothesized relationship between Classroom Environment and Attitudes Towards Math is not statistically significant.
- Class Environment and Home Environment do not effect Standardized testing directly.
- Home Environment significantly (0.636) effects Standardized testing indirectly.

(iii) Outer model reliability and validity

Next the correlations between the latent variable and the indicators in the outer model will be examined. Firstly, looking at the **Stop Criterion** function in

SMART-PLS it is shown that the algorithm converged after only 8 iterations which is much better than the default 300. This is a quick confirmation that the model is not completely wrong.

The **Outer Loadings** function provides information on the path coefficients, internal consistency, and internal validity.

TABLE 4. Results Summary for Reflective Outer Models and Inner Models

Latent Variable	Indicators	Loadings	Indicator Reliability (i.e. $loadings^2$)	Cronbach's Alpha	AVE
Class Environment	class size;	-0.274	0.075	0.889	0.282
	fulltime;	-0.330	0.108		
	frl;	0.621	0.385		
	ell;	0.488	0.238		
	min;	0.472	0.222		
	sped;	0.813	0.660		
Home Environment	ps;	0.998	0.9960		
	avg.income;	0.241	0.058		
Attitude Towards Math	ss;	1.000	1.000	1.000	1.000
Standardized Testing Proficiency	tcapmath;	0.951	0.904		
	tcapread/write;	0.947	0.896		

(iv) **Indicator Significance and Reliability**

The recommended minimum significance value is 0.5. It can be seen that *classsize*, *fulltime*, and *avg.income* are not statistically significant indicators. The recommended minimum reliability value is 0.4 and the preferred level is 0.7. It can be seen that within the Class Environment latent variable that *frl* and *sped* approach or exceed the minimum reliability value. This information suggests that the Class Environment some indicators demonstrate a low level of significance and internal consistency reliability. This may suggest that there are too many formative indicators for this variable. Within the Home Environment only *ps* indicator exceeds the preferred level values. All indicators within the Attitudes Towards Math and Standardized Testing Proficiency exceed the preferred level values.

(v) **Cronbach's Alpha**

Cronbach's Alpha is used to measure internal consistency reliability. The values for both Class Environment and Attitudes Towards Math exceed the preferred level of 0.7 thus they have high levels of internal consistency reliability. However, Standardized Testing Proficiency and Home Environment do not.

(vi) **Convergent Validity** In order to confirm convergence validity, each latent variable's Average Variance Extracted (AVE) values are looked at. From table 2, it is shown that Home Environment and Standardized Testing Proficiency do not have AVE values. Attitudes Towards Math AVE value is greater than the preferred threshold of 0.5. Thus the convergent validity of Attitudes Towards Math is confirmed.

(v) **Discriminant validity**

In a 1981 Journal of Marketing Research, Fornell and Larcker suggested that if the square root of a latent variable's AVE values is greater than other correlation values among latent variables than the value can be used to confirm discriminant validity.

TABLE 5. Fornell-Larcker Criterion Analysis for Checking Discriminant Validity

	Attitude Toward Math	Class Environment	Home Environment	Standardized Testing Proficiency
Attitude Toward Math	Single Construct Variable			
Class Environment	-0.392	0.531		
Home Environment	0.716	-0.320		
Standardized Testing Proficiency	0.762	-0.485	0.351	

For example, the latent variable Classroom Environment's AVE is demonstrated to be 0.282 from table 2. thus its square root is 0.531. Its correlation values in row and column of Classroom Environment's are (-0.392) and (-0.320, -0.485) respectively. The square root of Classroom Environment's AVE value is greater than the latent variable correlations. This indicates that discriminant validity is well established.

6. Further Testing

SMART-PLS is able to further examine the model from small sample sizes through methods such as bootstrapping and blindfolding. These processes re-sample a large number of sub-samples from the given data which estimates the procedures of sample statistics (mean, percentiles, etc.) and treats the re-sampled data as new. This has sparked controversy in the "truth" of the data. However, it is not the point of this paper to engage in the debate.

(vii) **Checking Structural Path Significance in Bootstrapping**

6.1. **Bootstrapping.** To obtain confirmation of the structural path significance, we use the bootstrapping function on SMART-PLS. Bootstrapping creates sub-samples with randomly selected observations from the original data. These sub-samples are then used to re-estimate the model. Bootstrapping gives mean values and standard errors for inner model path coefficients and weights and loadings in

the outer model. The disadvantages of bootstrapping also lie in its simplicity. The assumptions that are made in order to create sub-samples and reorder data are not explicitly stated. Thus, the number of bootstrap samples recommended has increased greatly with advanced computing power as to avoid random sampling errors.

TABLE 6. Inner Path Significance

Linkage #		
1	Attitude Towards Math \Rightarrow Standardized Test Proficient	5.920
2	Class Environment \Rightarrow Attitudes Towards Math	0.765
3	Class Environment \Rightarrow Standardized Test Proficient	0.873
4	Home Environment \Rightarrow Attitudes Towards Math	5.924
5	Home Environment \Rightarrow Standardized Test Proficient	1.739

Using the two-tailed t-test with 5% significance level, we can test path significance if the T-statistics is larger than 1.96. In examining the inner model, it can be seen that the 2, 3, and 5 linkages are not significant.

After checking inner path significance using Bootstrapping, the outer loadings can be explored. Examining the T-statistic of the outer loadings, the same standard of 1.96 is used to check for high significance.

TABLE 7. Outer Path Significance

	Classroom Environment	Home Environment	Attitudes Towards Math	Standardized Testing Proficiency
ell;	2.802			
frl;	2.972			
sped;	2.775			
min;	3.087			
classsize;	0.088			
fulltime;	2.342			
ps;		10.393		
avg.income;		0.724		
ss;			Single Item Construct	
tcap1;				16.344
tcap2;				13.081

It can be seen that the *classroom size* and *avg. income* indicators have low T-statistic values which indicates that they do not have a high significance in the outer model loadings. All other indicators have high significance. These findings support the indicator reliability findings above.

7. Blindfolding

Blindfolding is an iterative process that calculates the Q^2 criterion which evaluates how well the model can predict the data of the cases that are omitted. Q^2 refers to predictive relevance. The iterative process involved omitting one case at a time and then re-estimating the model. The omitted values are then predicted by the re-estimated model. The Q^2 value in the cross validated redundancy test estimates the predictive relevance of target endogenous variables. A model has predictive relevance if $Q^2 > 0$ and is lacking if $Q^2 \approx 0$ or $Q^2 < 0$.

TABLE 8. Cross-validated Redundancy Values Q^2

	SSO	SSE	Q^2
Attitudes Towards Math	27.00	15.804	0.415
Standardized Testing Proficiency	54.00	23.810	0.559

It can be seen that predictive relevance is demonstrated by both endogenous latent variables.

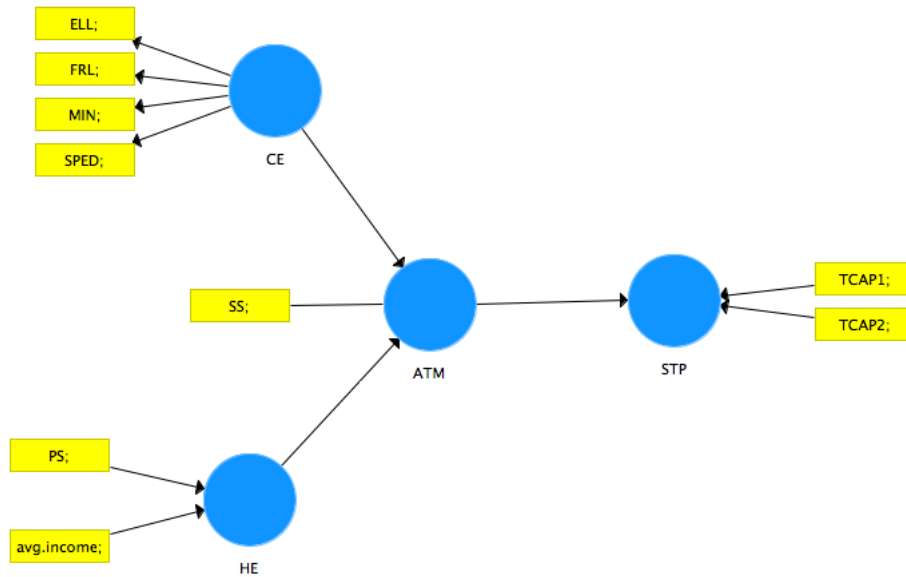
8. Implications

The real world implications of this study are manifold. First, we will address the insignificant findings. The insignificance of the indicators *class size* and *avg.income* was surprising. These findings may have been due to taking the average from each high school for these values making the values relatively similar for all schools. Perhaps, a better way to examine these indicators would be to look at each individual class and parental income within one school. The path from Class Environment to Standardized Testing Proficiency was found to be weak due to insignificant and inconsistent indicators, not necessarily due to a falsely hypothesized relationship. The hypothesized direct relationship between Home Environment and Standardized Testing is shown to be false. We learned that Class Environment and Home Environment explain the 54.2% variance in Attitudes Towards Mathematics. This is significant considering how many other variables could affect an attitude for example what one's friends think about math, what period is math offered, or if the classroom is air conditioned or not. We learned that attitudes strongly affects standardized testing proficiency which is a reasonable conclusion. A student who has a positive attitude towards a subject will perform better on a test than one who does not. SEM evaluates how significantly indicators affect latent variables and latent variables affect each other but it does not conclude if the effects are positive or negative. The results of this study could be used by charter schools to determine if the findings affect attitudes for the better or not.

9. Conclusion and Re-specification

Understanding SEM allows one to measure un-measurable constructs, opening exciting doors for explaining phenomena in our world that cannot be measured. SEM was best option for this study considering how the model was constructed, using hypothesis and observed relationships as the primary point of departure. The confirmatory nature of SEM allowed the study to test the model that was believed to be true for its validity and to evaluate relationships. This allowed the study to focus on and to confirm or not its chosen areas of interest. The weaknesses of the study lie in its small sample size, the data available, and the construction of the model. Much of the data was affected by the averaging as mentioned above. The SPF report card that noted student satisfaction/engagement and parent satisfaction/encouragement was a self-reported evaluation which may have affected the validity of the study. The study would have benefited from an evaluation of parent and student opinions on math specifically. The latent construct "attitudes towards mathematics" logically is the target latent variable by the construction of the study's hypothesized relationships however the construct is not necessarily implied by the relationships. The SMART-PLS program only allows for single headed arrows which limited correlation relationships between variables. To further this study, one could account for the many other variables that could possibly affect "attitudes towards math" and their underlying constructs. The study could be expanded to include more high schools or could be explored in one high school. To leave our readers on a note that looks to the future of math education, we have included the re-specified model.

10. Re-Specified Model



11. References

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NOTES

¹Abraham Lincoln High School, Bruce Randolph High School, CEC Middle College of Denver, Collegiate Preparatory Academy, Denver School of the Arts, DCIS at Montebello, Denver Center for International Studies, DSST at Green Valley Ranch, DSST at Stapleton, East High School, George Washington High School, John F. Kennedy High School, KIPP Denver Collegiate High School, Manual High School, Martin Luther King High School, Noel Community Arts School, North H.S., Sims-Fayola International Academy, South H.S., Southwest Early College, STRIVE Prep - Smart, STRIVE Prep - Excel, Thomas Jefferson High School, Venture Preparatory High School, West Generations High School, West Leadership Academy.

²Herman Wold developed partial least squares path modeling in the 1960's. Since its creation, PLS has gone through many gradual changes and a vast evolutionary process that has brought the method to the high recognition it has today.

³In market research, a moderate R^2 value is 0.50

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